

Chapter 4

Understanding Interest Rates

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Present Value

- A dollar paid to you one year from now is less valuable than a dollar paid to you today



Discounting the Future

Let $i = .10$

In one year $\$100 \times (1 + 0.10) = \110

In two years $\$110 \times (1 + 0.10) = \121

or $100 \times (1 + 0.10)^2$

In three years $\$121 \times (1 + 0.10) = \133

or $100 \times (1 + 0.10)^3$

In n years

$\$100 \times (1 + i)^n$



Simple Present Value

PV = today's (present) value

CF = future cash flow (payment)

i = the interest rate

$$PV = \frac{CF}{(1 + i)^n}$$

A vertical column of silver coins is positioned on the left side of the slide, partially overlapping the blue background.

Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond



Yield to Maturity

- The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today



Simple Loan—Yield to Maturity

PV = amount borrowed = \$100

CF = cash flow in one year = \$110

n = number of years = 1

$$\$100 = \frac{\$110}{(1 + i)^1}$$

$$(1 + i) \$100 = \$110$$

$$(1 + i) = \frac{\$110}{\$100}$$

$$i = 0.10 = 10\%$$

For simple loans, the simple interest rate equals the yield to maturity



Fixed Payment Loan— Yield to Maturity

The same cash flow payment every period throughout
the life of the loan

LV = loan value

FP = fixed yearly payment

n = number of years until maturity

$$LV = \frac{FP}{1 + i} + \frac{FP}{(1 + i)^2} + \frac{FP}{(1 + i)^3} + \dots + \frac{FP}{(1 + i)^n}$$



Coupon Bond—Yield to Maturity

Using the same strategy used for the fixed-payment loan:

P = price of coupon bond

C = yearly coupon payment

F = face value of the bond

n = years to maturity date

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \dots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

TABLE 1

**Yields to Maturity on a 10%-Coupon-Rate Bond Maturing
in Ten Years (Face Value = \$1,000)**

Price of Bond (\$)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate
- The price of a coupon bond and the yield to maturity are negatively related
- The yield to maturity is greater than the coupon rate when the bond price is below its face value



Consol or Perpetuity

- A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

$$P_c = C / i_c$$

P_c = price of the consol

C = yearly interest payment

i_c = yield to maturity of the consol

Can rewrite above equation as $i_c = C / P_c$

For coupon bonds, this equation gives current yield—an easy-to-calculate approximation of yield to maturity



Discount Bond—Yield to Maturity

For any one year discount bond

$$i = \frac{F - P}{P}$$

F = Face value of the discount bond

P = current price of the discount bond

The yield to maturity equals the increase
in price over the year divided by the initial price.
As with a coupon bond, the yield to maturity is
negatively related to the current bond price.



Yield on a Discount Basis

Less accurate but less difficult to calculate

$$i_{db} = \frac{F - P}{F} \times \frac{360}{\text{days to maturity}}$$

i_{db} = yield on a discount basis

F = face value of the Treasury bill (discount bond)

P = purchase price of the discount bond

Uses the percentage gain on the face value

Puts the yield on an annual basis using 360 instead of 365 days

Always understates the yield to maturity

The understatement becomes more severe the longer the maturity

Following the Financial News: Bond Prices and Interest Rates

(a) Treasury bonds
and notes

	GOVT. BONDS & NOTES					
	Rate	Maturity Mo/Yr	Bid	Asked	Chg.	Ask Yld.
T-bond 1 —	1.875	Jan 06n	99:28	99:29	1	3.76
	5.625	Feb 06n	100:03	100:04	...	3.96
	9.375	Feb 06	100:14	100:15	...	3.98
	1.625	Feb 06n	99:21	99:22	...	4.07
	1.500	Mar 06n	99:12	99:13	...	4.24
	2.250	Apr 06n	99:11	99:12	...	4.32
	2.000	May 06n	99:06	99:07	1	4.34
	4.625	May 06n	100:01	100:02	...	4.36

(b) Treasury bills

TREASURY BILLS				
Maturity	Days to Mat.	Bid	Asked	Ask Yld.
Jan 19 06	6	4.01	4.00	-0.08 4.06
Jan 26 06	13	4.01	4.00	-0.04 4.06
T-bill 1 —	Feb 02 06	20	4.04	4.03 -0.03 4.10
	Feb 09 06	27	4.06	4.05 -0.02 4.12
	Feb 16 06	34	4.05	4.04 -0.02 4.11
	Feb 23 06	41	4.06	4.05 -0.02 4.13
	Mar 02 06	48	4.12	4.11 0.01 4.19
	Mar 09 06	55	4.13	4.12 0.01 4.20
	Mar 16 06	62	4.11	4.10 -0.01 4.19
	Mar 23 06	69	4.17	4.16 0.01 4.25
	Mar 30 06	76	4.19	4.18 0.02 4.28
	Apr 06 06	83	4.20	4.19 0.02 4.29
	Apr 13 06	90	4.21	4.20 0.01 4.30

Source: *Wall Street Journal*, Thursday, January 13, 2006.

(c) New York Stock Exchange bonds

Thursday, January 12, 2006

Forty most active fixed-coupon corporate bonds

Company (TICKER)	Coupon	Maturity	Last Price	Last Yield	*Est Spread	Ust†	Est \$ Vol (000's)
Tyco International Group (TYC)	6.000	Nov 15, 2013	103.104	5.505	110	10	143,116
HSBC Finance Corp (HSBC)	5.250	Jan 14, 2011	100.169	5.211	83	5	101,312
Wells Fargo (WFC)	4.875	Jan 12, 2011	99.851	4.909	54	5	94,719
Bank of America Corp (BAC)	4.875	Sep 15, 2012	99.217	5.014	57	5	93,623
Verizon New York Inc (VZ)	7.000	Dec 01, 2033	97.625	7.198	261	30	85,530

Volume represents total volume for each issue; price/yield data are for trades of \$1 million and greater. *Estimated spreads, in basis points (100 basis points is one percentage point), over the 2, 3, 5, 10, or 30-year hot run Treasury note/bond. 2-year: 4.375 12/07; 3-year: 4.375 11/08; 5-year: 4.375 12/10; 10-year: 4.500 11/15; 30-year: 5.375 02/31. †Comparable U.S. Treasury issue.

Source: MarketAxess Corporate BondTicker



Rate of Return

The payments to the owner plus the change in value
expressed as a fraction of the purchase price

$$\text{RET} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

RET = return from holding the bond from time t to time $t + 1$

P_t = price of bond at time t

P_{t+1} = price of the bond at time $t + 1$

C = coupon payment

$\frac{C}{P_t}$ = current yield = i_c

$\frac{P_{t+1} - P_t}{P_t}$ = rate of capital gain = g



Rate of Return and Interest Rates

- The return equals the yield to maturity only if the holding period equals the time to maturity
- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period
- The more distant a bond's maturity, the greater the size of the percentage price change associated with an interest-rate change



Rate of Return and Interest Rates (cont'd)

- The more distant a bond's maturity, the lower the rate of return the occurs as a result of an increase in the interest rate
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise

TABLE 2

**One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds
When Interest Rates Rise from 10% to 20%**

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price (\$)	(4) Price Next Year*	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	-49.7	-39.7
20	10	1,000	516	-48.4	-38.4
10	10	1,000	597	-40.3	-30.3
5	10	1,000	741	-25.9	-15.9
2	10	1,000	917	-8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.



Interest-Rate Risk

- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds
- There is no interest-rate risk for any bond whose time to maturity matches the holding period



Real and Nominal Interest Rates

- Nominal interest rate makes no allowance for inflation
- Real interest rate is adjusted for changes in price level so it more accurately reflects the cost of borrowing
- Ex ante real interest rate is adjusted for expected changes in the price level
- Ex post real interest rate is adjusted for actual changes in the price level



Fisher Equation

$$i = i_r + \pi^e$$

i = nominal interest rate

i_r = real interest rate

π^e = expected inflation rate

When the real interest rate is low,
there are greater incentives to borrow and fewer incentives to lend.

The real interest rate is a better indicator of the incentives to
borrow and lend.

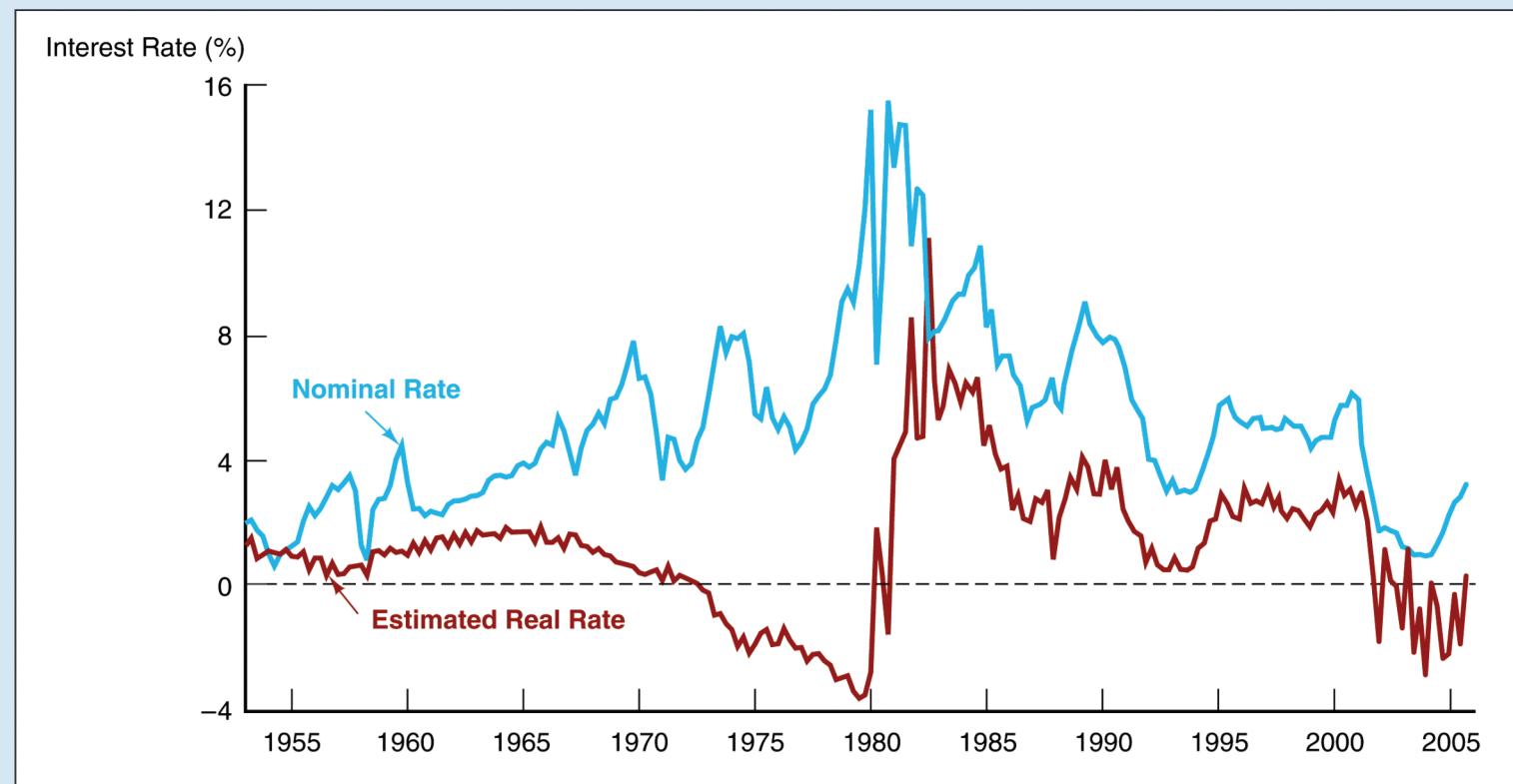


FIGURE 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2005

Sources: Nominal rates from www.federalreserve.gov/releases/H15. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie-Rochester Conference Series on Public Policy* 15 (1981): 151–200. This procedure involves estimating expected inflation as a function of past interest rates, inflation, and time trends and then subtracting the expected inflation measure from the nominal interest rate.