Chapter 4

Understanding Interest Rates
Present Value

• A dollar paid to you one year from now is less valuable than a dollar paid to you today
Discounting the Future

Let $i = .10$

In one year  $100 \times (1 + 0.10) = $110
In two years  $110 \times (1 + 0.10) = $121
or $100 \times (1 + 0.10)^2$
In three years  $121 \times (1 + 0.10) = $133
or $100 \times (1 + 0.10)^3$

In $n$ years

$100 \times (1 + i)^n$
Simple Present Value

\[ PV = \text{today's (present) value} \]
\[ CF = \text{future cash flow (payment)} \]
\[ i = \text{the interest rate} \]

\[ PV = \frac{CF}{(1 + i)^n} \]
Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond
Yield to Maturity

- The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today
Simple Loan—Yield to Maturity

PV = amount borrowed = $100
CF = cash flow in one year = $110

\[ n = \text{number of years} = 1 \]

\[ 100 = \frac{110}{(1 + i)^1} \]

\[ (1 + i) \times 100 = 110 \]

\[ (1 + i) = \frac{110}{100} \]

\[ i = 0.10 = 10\% \]

For simple loans, the simple interest rate equals the yield to maturity
Fixed Payment Loan—Yield to Maturity

The same cash flow payment every period throughout the life of the loan

\[ LV = \text{loan value} \]
\[ FP = \text{fixed yearly payment} \]
\[ n = \text{number of years until maturity} \]

\[ LV = \frac{FP}{1 + i} + \frac{FP}{(1 + i)^2} + \frac{FP}{(1 + i)^3} + \ldots + \frac{FP}{(1 + i)^n} \]
Coupon Bond—Yield to Maturity

Using the same strategy used for the fixed-payment loan:

\[ P = \text{price of coupon bond} \]

\[ C = \text{yearly coupon payment} \]

\[ F = \text{face value of the bond} \]

\[ n = \text{years to maturity date} \]

\[ P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n} \]
When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.

The price of a coupon bond and the yield to maturity are negatively related.

The yield to maturity is greater than the coupon rate when the bond price is below its face value.

### Table 1

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>
Consol or Perpetuity

- A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

\[ P_c = \frac{C}{i_c} \]

- \( P_c \) = price of the consol
- \( C \) = yearly interest payment
- \( i_c \) = yield to maturity of the consol

Can rewrite above equation as \( i_c = \frac{C}{P_c} \)

For coupon bonds, this equation gives current yield—an easy-to-calculate approximation of yield to maturity
Discount Bond—Yield to Maturity

For any one year discount bond

\[ i = \frac{F - P}{P} \]

\( F = \text{Face value of the discount bond} \)
\( P = \text{current price of the discount bond} \)

The yield to maturity equals the increase in price over the year divided by the initial price. As with a coupon bond, the yield to maturity is negatively related to the current bond price.
Yield on a Discount Basis

Less accurate but less difficult to calculate

\[ i_{db} = \frac{F - P}{F} \times \frac{360}{\text{days to maturity}} \]

\( i_{db} \) = yield on a discount basis

F = face value of the Treasury bill (discount bond)

P = purchase price of the discount bond

Uses the percentage gain on the face value

Puts the yield on an annual basis using 360 instead of 365 days

Always understates the yield to maturity

The understatement becomes more severe the longer the maturity
# Following the Financial News: Bond Prices and Interest Rates

## (a) Treasury bonds and notes

<table>
<thead>
<tr>
<th>Rate</th>
<th>Maturity Mo/Yr</th>
<th>Bid</th>
<th>Asked</th>
<th>Chg.</th>
<th>Ask Yld.</th>
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</thead>
<tbody>
<tr>
<td>1.875</td>
<td>Jan 06n</td>
<td>99:28</td>
<td>99:29</td>
<td>1</td>
<td>3.76</td>
</tr>
<tr>
<td>9.625</td>
<td>Feb 06n</td>
<td>100:04</td>
<td>100:04</td>
<td>...</td>
<td>3.06</td>
</tr>
<tr>
<td>9.375</td>
<td>Feb 06n</td>
<td>100:14</td>
<td>100:15</td>
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<td>3.88</td>
</tr>
<tr>
<td>1.625</td>
<td>Feb 06n</td>
<td>99:21</td>
<td>99:22</td>
<td>...</td>
<td>4.07</td>
</tr>
<tr>
<td>1.500</td>
<td>Mar 06n</td>
<td>99:12</td>
<td>99:13</td>
<td>...</td>
<td>4.24</td>
</tr>
<tr>
<td>2.250</td>
<td>Apr 06n</td>
<td>99:11</td>
<td>99:12</td>
<td>...</td>
<td>4.32</td>
</tr>
<tr>
<td>2.000</td>
<td>May 06n</td>
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<td>99:07</td>
<td>1</td>
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<tr>
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<td>...</td>
<td>4.36</td>
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## (b) Treasury bills

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
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<td>Jan 19 06</td>
<td>6 4.01</td>
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<td>20 4.04</td>
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<td>Feb 09 06</td>
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<td>4.05</td>
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<tr>
<td>Feb 16 06</td>
<td>34 4.05</td>
<td>4.04</td>
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<tr>
<td>Feb 23 06</td>
<td>41 4.06</td>
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<td>-0.02</td>
<td>4.13</td>
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<td>0.01</td>
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<tr>
<td>Mar 09 06</td>
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<td>4.12</td>
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<td>4.20</td>
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<tr>
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<td>62 4.11</td>
<td>4.10</td>
<td>-0.01</td>
<td>4.19</td>
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<tr>
<td>Mar 23 06</td>
<td>69 4.17</td>
<td>4.16</td>
<td>0.01</td>
<td>4.25</td>
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<tr>
<td>Mar 30 06</td>
<td>76 4.19</td>
<td>4.18</td>
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<td>4.28</td>
</tr>
<tr>
<td>Apr 06 06</td>
<td>83 4.20</td>
<td>4.19</td>
<td>0.02</td>
<td>4.29</td>
</tr>
<tr>
<td>Apr 13 06</td>
<td>90 4.21</td>
<td>4.20</td>
<td>0.01</td>
<td>4.30</td>
</tr>
</tbody>
</table>


## (c) New York Stock Exchange bonds

### CORPORATE BONDS

<table>
<thead>
<tr>
<th>Company (TICKER)</th>
<th>Coupon</th>
<th>Maturity</th>
<th>Last Price</th>
<th>Last Yield</th>
<th>Est $ Spread</th>
<th>Est $ Vol (000's)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yipo International Group (TYC)</td>
<td>8,000</td>
<td>Nov 15, 2013</td>
<td>103,314</td>
<td>5,505</td>
<td>110</td>
<td>10</td>
</tr>
<tr>
<td>HSBC Finance Corp (HSBC)</td>
<td>5,250</td>
<td>Jan 14, 2011</td>
<td>100,169</td>
<td>5,211</td>
<td>83</td>
<td>5</td>
</tr>
<tr>
<td>Wells Fargo (WFC)</td>
<td>4,875</td>
<td>Jan 12, 2011</td>
<td>99,851</td>
<td>4,909</td>
<td>54</td>
<td>5</td>
</tr>
<tr>
<td>Bank of America Corp (BAC)</td>
<td>4,875</td>
<td>Sep 15, 2012</td>
<td>99,217</td>
<td>5,014</td>
<td>57</td>
<td>5</td>
</tr>
<tr>
<td>Verizon New York Inc (VZ)</td>
<td>7,000</td>
<td>Dec 01, 2033</td>
<td>97,625</td>
<td>7,198</td>
<td>261</td>
<td>30</td>
</tr>
</tbody>
</table>

Volume represents total volume for each issue; price/yield data are for trades of $1 million and greater. *Estimated spreads, in basis points (100 basis points is one percentage point), over the 2, 3, 5, 10, or 30-year Treasury note/bond. 2-year: 4.375 12/07; 3-year: 4.575 12/08; 5-year: 4.675 12/10; 7-year: 4.675 06/12; 10-year: 4.500 11/15; 30-year: 5.375 02/31.† Comparable U.S. Treasury issue.

Source: MarketAxess Corporate BondTicker
Rate of Return

The payments to the owner plus the change in value expressed as a fraction of the purchase price

$$\text{RET} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

RET = return from holding the bond from time $t$ to time $t + 1$

$P_t$ = price of bond at time $t$

$P_{t+1}$ = price of the bond at time $t + 1$

$C$ = coupon payment

$$\frac{C}{P_t} = \text{current yield } = i_c$$

$$\frac{P_{t+1} - P_t}{P_t} = \text{rate of capital gain } = g$$
Rate of Return and Interest Rates

- The return equals the yield to maturity only if the holding period equals the time to maturity.
- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period.
- The more distant a bond’s maturity, the greater the size of the percentage price change associated with an interest-rate change.
Rate of Return and Interest Rates (cont’d)

• The more distant a bond’s maturity, the lower the rate of return the occurs as a result of an increase in the interest rate.

• Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise.
<table>
<thead>
<tr>
<th>Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return (2 + 5) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>−49.7</td>
<td>−39.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>1,000</td>
<td>516</td>
<td>−48.4</td>
<td>−38.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1,000</td>
<td>597</td>
<td>−40.3</td>
<td>−30.3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1,000</td>
<td>741</td>
<td>−25.9</td>
<td>−15.9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1,000</td>
<td>917</td>
<td>−8.3</td>
<td>+1.7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated using Equation 3.
Interest-Rate Risk

• Prices and returns for long-term bonds are more volatile than those for shorter-term bonds

• There is no interest-rate risk for any bond whose time to maturity matches the holding period
Real and Nominal Interest Rates

• Nominal interest rate makes no allowance for inflation
• Real interest rate is adjusted for changes in price level so it more accurately reflects the cost of borrowing
• Ex ante real interest rate is adjusted for expected changes in the price level
• Ex post real interest rate is adjusted for actual changes in the price level
Fisher Equation

\[ i = i_r + \pi^e \]

- \( i \) = nominal interest rate
- \( i_r \) = real interest rate
- \( \pi^e \) = expected inflation rate

When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend. The real interest rate is a better indicator of the incentives to borrow and lend.
**Figure 1** Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2005