Combinatorics Formula Sheet

Factorial

factorial of a non-negative integer n : $n! = n(n-1)(n-2)\cdots 3 \bullet 2 \bullet 1$

special case : $0! = 1$

Combinations

n Different Objects Taken r Objects at a Time : $${}_nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Permutations

n Different Objects : $n!$

n Different Objects Taken r Objects at a Time : (1) $${}_nP_r = \frac{n!}{(n-r)!}$$

(2) $${}_nP_r = n(n-1)(n-2)\cdots(n-r+1)$$

n Objects Not All Different (Distinguishable P's) : $$\frac{n!}{n_1!\,n_2!\,n_3!\,\cdots\,n_k!}$$

n Different Objects arranged in a Circle : $(n-1)!$

Binomial Expansion

1. The number of the terms in the expansion of $(a+b)^n$ is $n+1$.
2. The coefficient of the first term is 1.
3. The coefficient of any other term is the product of the coefficient of the preceding term and the exponent of a in the preceding term divided by the number of the preceding term.
4. The exponent of a in any term after the first term is one less than the exponent of a in the preceding term. (The powers of a decrease from n to 0.)
5. The exponent of b in any term after the first term is one greater than the exponent of b in the preceding term. (The powers of b increase from 0 to n .)
6. The sum of the exponents of a and b in each term is n .

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + \binom{n}{n}b^n$$

Alternate Form :

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \qquad \text{where } \binom{n}{r} = {}_nC_r = \frac{n!}{r!\,(n-r)!}$$

k -th term Formula :

k-th term of $(a+b)^n$ is $\binom{n}{k-1}a^{n-(k-1)}b^{k-1}$

Row	*Pascal's Triangle*
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1
$\vdots$	$\vdots$

The Inclusion-Exclusion Principle

For any two sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

The Complement Principle

If set A is a subset of a universal set U, then $n(A) = n(U) - n(A^C)$.