HOMEWORK 5

1) Let $X_1, X_2, \dots, X_n$ be an independent random sample from a distribution with pdF

$$f(x;\theta) = \begin{cases} \theta\, 2^{-\theta}\, x^{\theta-1}, & 0<x<2 \\ 0, & \text{otherwise} \end{cases}$$

Find the limiting distribution of $\frac{n^{1/\theta}\, Y_{(1)}}{2}$ where $Y_{(1)}$ is the first order statistic.

2) If $X_1, X_2, \dots, X_n$ be a random sample with $X_i \sim Exp(1)$ find the limiting distribution of $Y=\sqrt{n}\,(\bar{X}-1)$ where ($\bar{X}$ is the mean of $X_1, \dots, X_n$.)

(Hint: First find $E(X)$ and $var(X)$ and then use central limit theorem)

3) If we have $X_1, \dots, X_n$ iid random variable from a pdF

$$f(x;\theta) = \frac{x^3}{6\theta^4}\, e^{-\theta/x} \quad \text{where } x>0 \quad \theta>0$$

with $E(X)=4\theta$ and $var(X)=4\theta^2$

a) Find $\hat{\theta}_1$ the method of moment estimator of θ.

b) Find $\hat{\theta}_2$ the maximum likelihood estimator of θ.

c) Evaluate the consistency and unbiasedness of $\hat{\theta}_2$ (MLE est.).

4) Show that the family of distribution Binomial$(n=2, p)$ $0<p<1$ is complete.

5) Let $X_1, \dots, X_n$ is a random sample from a distribution with pdF

$$f(x;\theta) = \begin{cases} \frac{2x}{\theta^2} & 0<x<\theta \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $\hat{\theta} = \frac{3}{2}\bar{X}$ is an unbiased estimator of θ

b) Evaluate the efficiency of $\hat{\theta}$ by finding CRLB (Cramer Rao lower bound).