ENERGY & ENVIRONMENT A type of exergetic efficiency known as the *well-to-wheel efficiency* is used to compare different options for powering vehicles. The calculation of this efficiency begins at the well where the oil or natural gas feedstock is extracted from the ground and ends with the power delivered to a vehicle's wheels. The efficiency accounts separately for how effectively the vehicle's fuel is produced from feedstock, called the *well-to-fuel tank efficiency,* and how effectively the vehicle's power plant converts its fuel to power, called the *fuel tank-to-wheel efficiency.* The product of these gives the *overall* well-to-wheel efficiency.

The table below gives sample well-to-wheel efficiency values for three power plant options as reported by an automobile manufacturer:

	Well-to-Tank (Fuel Production Efficiency) (%)	×	Tank-to-Wheel (Vehicle Efficiency) (%)	=	Well-to-Wheel (Overall Efficiency) (%)
Conventional gasoline-fueled engine	88	×	16	=	14
Hydrogen-fueled fuel cell[a]	58	×	38	=	22
Gasoline-fueled hybrid electric	88	×	32	=	28

[a]Hydrogen produced from natural gas.

These data show that vehicles using conventional internal combustion engines do not fare well in terms of the well-to-wheel efficiency. The data also show that fuel-cell vehicles operating on hydrogen have the best tank-to-wheel efficiency of the three options, but lose out on an overall basis to hybrid vehicles, which enjoy a higher well-to-tank efficiency. Still, the well-to-wheel efficiency is just one consideration when making policy decisions concerning different options for powering vehicles. With increasing concern over global atmospheric CO_2 concentrations, another consideration is the well-to-wheel *total* production of CO_2 in kg per km driven (lb per mile driven).

7.7 Thermoeconomics

Thermal systems typically experience significant work and/or heat interactions with their surroundings, and they can exchange mass with their surroundings in the form of hot and cold streams, including chemically reactive mixtures. Thermal systems appear in almost every industry, and numerous examples are found in our everyday lives. Their design and operation involve the application of principles from thermodynamics, fluid mechanics, and heat transfer, as well as such fields as materials, manufacturing, and mechanical design. The design and operation of thermal systems also require explicit consideration of engineering economics, for cost is always a consideration. The term **thermoeconomics** may be applied to this general area of application, although it is often applied more narrowly to methodologies combining exergy and economics for optimization studies during design of new systems and process improvement of existing systems.

thermoeconomics

7.7.1 Costing

Is costing an art or a science? The answer is a little of both. *Cost engineering* is an important engineering subdiscipline aimed at objectively applying real-world costing experience in engineering design and project management. Costing services are provided by practitioners skilled in the use of specialized methodologies, cost models, and databases, together with costing expertise and judgment garnered from years of professional practice. Depending on need, cost engineers provide services ranging from rough and rapid estimates to in-depth analyses. Ideally, cost engineers are involved with projects from the formative stages, for the *output* of cost engineering is an essential *input* to decision making. Such input can be instrumental in identifying feasible options from a set of alternatives and even pinpointing the best option.

Costing of thermal systems considers costs of owning and operating them. Some observers voice concerns that costs related to the environment often are only weakly taken into consideration in such evaluations. They say companies pay for the right to extract natural resources used in the production of goods and services but rarely pay fully for depleting nonrenewable resources and mitigating accompanying environmental degradation and loss of wildlife habitat, in many cases leaving the cost burden to future generations. Another concern is who pays for the costs of controlling air and water pollution, cleaning up hazardous wastes, and the impacts of pollution and waste on human health—industry, government, the public, or some combination of each? Yet when agreement about environmental costs is achieved among interested business, governmental, and advocacy groups, such costs are readily integrated in costing of thermal systems, including costing on an exergy basis, which is the present focus.

7.7.2 Using Exergy in Design

To illustrate the use of exergy reasoning in design, consider Fig. 7.12 showing a boiler at steady state. Fuel and air enter the boiler and react to form hot combustion gases.

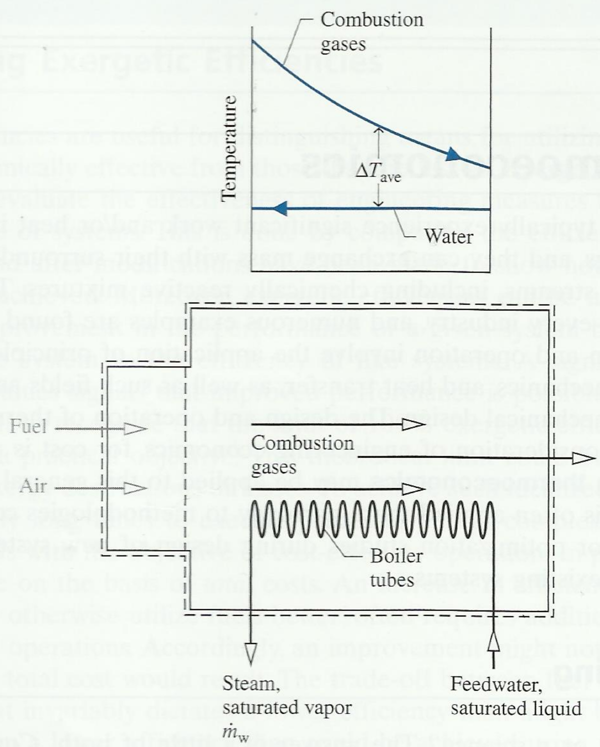

Fig. 7.12 Boiler used to discuss exergy in design.

Feedwater also enters as saturated liquid, receives exergy by heat transfer from the combustion gases, and exits without temperature change as saturated vapor at a specified condition for use elsewhere. Temperatures of the hot gas and water streams are also shown on the figure.

There are two main sources of exergy destruction in the boiler: (1) irreversible heat transfer occurring between the hot combustion gases and the water flowing through the boiler tubes and (2) the combustion process itself. To simplify the present discussion, the boiler is considered to consist of a combustor unit in which fuel and air are burned to produce hot combustion gases, followed by a heat exchanger unit where water is vaporized as the hot gases cool.

The present discussion centers on the heat exchanger unit. Let us think about its total cost as the sum of fuel-related and capital costs. We will also take the average temperature difference between the two streams, ΔT_{ave}, as the *design variable*. From our study of the second law of thermodynamics, we know that the average temperature difference between the two streams is a measure of exergy destruction associated with heat transfer between them. The exergy destroyed owing to heat transfer originates in the fuel entering the boiler. Accordingly, a cost related to fuel consumption can be attributed to this source of irreversibility. Since exergy destruction increases with temperature difference between the streams, the fuel-related cost increases with *increasing* ΔT_{ave}. This variation is shown in Fig. 7.13 on an *annualized* basis, in dollars per year.

From our study of heat transfer, we know an inverse relation exits between ΔT_{ave} and the boiler tube surface area required for a desired heat transfer rate between the streams. For example, if we design for a small average temperature difference to reduce exergy destruction within the heat exchanger, this dictates a large surface area and typically a more costly boiler. From such considerations, we infer that boiler capital cost increases with *decreasing* ΔT_{ave}. This variation is shown in Fig. 7.13, again on an annualized basis.

The *total cost* is the sum of the capital cost and the fuel cost. The total cost curve shown in Fig. 7.13 exhibits a minimum at the point labeled a. Notice, however, that the curve is relatively flat in the neighborhood of the minimum, so there is a range of ΔT_{ave} values that could be considered *nearly optimal* from the standpoint of minimum total cost. If reducing the fuel cost were deemed more important than minimizing the capital cost, we might choose a design that would operate at point a′. Point a″ would

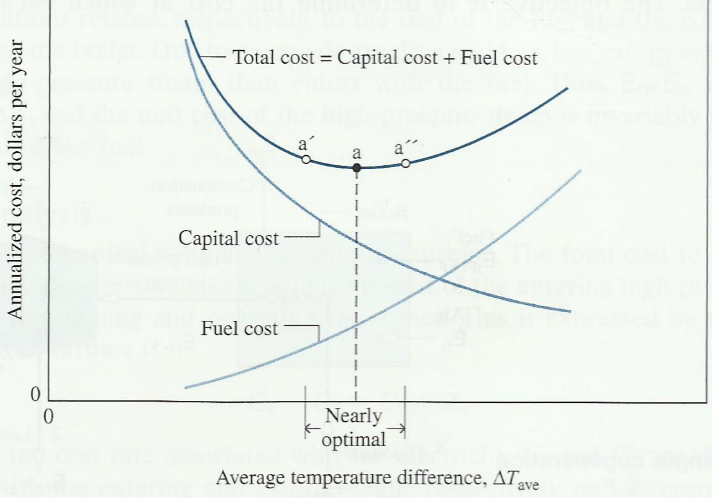

Fig. 7.13 Cost curves for the heat exchanger unit of the boiler of Fig. 7.12.

be a more desirable operating point if capital cost were of greater concern. Such trade-offs are common in design situations.

The actual design process differs significantly from the simple case considered here. For one thing, costs cannot be determined as precisely as implied by the curves in Fig. 7.13. Fuel prices vary widely over time, and equipment costs may be difficult to predict as they often depend on a bidding procedure. Equipment is manufactured in discrete sizes, so the cost also would not vary continuously as shown in the figure. Furthermore, thermal systems usually consist of several components that interact with one another. Optimization of components individually, as considered for the heat exchanger unit of the boiler, does not guarantee an optimum for the overall system. Finally, the example involves only ΔT_{ave} as a design variable. Often, several design variables must be considered and optimized simultaneously.

7.7.3 Exergy Costing of a Cogeneration System

Another important aspect of thermoeconomics is the use of exergy for *allocating* costs to the products of a thermal system. This involves assigning to each product the total cost to produce it, namely the cost of fuel and other inputs plus the cost of owning and operating the system (e.g., capital cost, operating and maintenance costs). Such costing is a common problem in plants where utilities such as electrical power, chilled water, compressed air, and steam are generated in one department and used in others. The plant operator needs to know the cost of generating each utility to ensure that the other departments are charged properly according to the type and amount of each utility used. Common to all such considerations are fundamentals from engineering economics, including procedures for annualizing costs, appropriate means for allocating costs, and reliable cost data.

To explore further the costing of thermal systems, consider the simple *cogeneration system* operating at steady state shown in Fig. 7.14. The system consists of a boiler and a turbine, with each having no significant heat transfer to its surroundings. The figure is labeled with exergy transfer rates associated with the flowing streams, where the subscripts F, a, P, and w denote fuel, combustion air, combustion products, and feedwater, respectively. The subscripts 1 and 2 denote high- and low-pressure steam, respectively. Means for evaluating the exergies of the fuel and combustion products are introduced in Chap. 13. The cogeneration system has two principal products: electricity, denoted by $\dot{W}_e$, and low-pressure steam for use in some process. The objective is to determine the cost at which each product is generated.

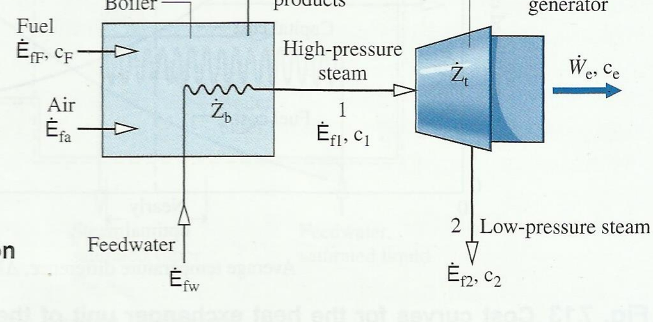

Fig. 7.14 Simple cogeneration system.

Boiler Analysis

Let us begin by evaluating the cost of the high-pressure steam produced by the boiler. For this, we consider a control volume enclosing the boiler. Fuel and air enter the boiler separately and combustion products exit. Feedwater enters and high-pressure steam exits. The total cost to produce the exiting high-pressure steam equals the total cost of the entering streams plus the cost of owning and operating the boiler. This is expressed by the following **cost rate balance** for the boiler

$$\dot{C}_1 = \dot{C}_F + \dot{C}_a + \dot{C}_w + \dot{Z}_b \tag{7.30}$$

cost rate balance

where $\dot{C}$ is the cost rate of the respective stream (in \$ per hour, for instance). $\dot{Z}_b$ accounts for the cost rate associated with owning and operating the boiler, including expenses related to proper disposal of the combustion products. In the present discussion, the cost rate $\dot{Z}_b$ is presumed known from a previous economic analysis.

Although the cost rates denoted by $\dot{C}$ in Eq. 7.30 are evaluated by various means in practice, the present discussion features the use of exergy for this purpose. Since exergy measures the true thermodynamic values of the work, heat, and other interactions between a system and its surroundings as well as the effect of irreversibilities within the system, exergy is a rational basis for assigning costs. With exergy costing, each of the cost rates is evaluated in terms of the associated rate of exergy transfer and a *unit cost.* Thus, for an entering or exiting stream, we write

$$\dot{C} = c\dot{E}_f \tag{7.31}$$

where c denotes the **cost per unit of exergy** (in \$ or cents per kW · h, for example) and $\dot{E}_f$ is the associated exergy transfer rate.

exergy unit cost

For simplicity, we assume the feedwater and combustion air enter the boiler with negligible exergy and cost. Thus Eq. 7.30 reduces as follows:

$$\dot{C}_1 = \dot{C}_F + \cancelto{0}{\dot{C}_a} + \cancelto{0}{\dot{C}_w} + \dot{Z}_b$$

Then, with Eq. 7.31 we get

$$c_1\dot{E}_{f1} = c_F\dot{E}_{fF} + \dot{Z}_b \tag{7.32a}$$

Solving for c_1, the unit cost of the high-pressure steam is

$$c_1 = c_F\left(\frac{\dot{E}_{fF}}{\dot{E}_{f1}}\right) + \frac{\dot{Z}_b}{\dot{E}_{f1}} \tag{7.32b}$$

This equation shows that the unit cost of the high-pressure steam is determined by two contributions related, respectively, to the cost of the fuel and the cost of owning and operating the boiler. Due to exergy destruction and loss, less exergy exits the boiler with the high-pressure steam than enters with the fuel. Thus, $\dot{E}_{fF}/\dot{E}_{f1}$ is invariably greater than 1, and the unit cost of the high-pressure steam is invariably greater than the unit cost of the fuel.

Turbine Analysis

Next, consider a control volume enclosing the turbine. The total cost to produce the electricity and low-pressure steam equals the cost of the entering high-pressure steam plus the cost of owning and operating the device. This is expressed by the *cost rate balance* for the turbine

$$\dot{C}_e + \dot{C}_2 = \dot{C}_1 + \dot{Z}_t \tag{7.33}$$

where $\dot{C}_e$ is the cost rate associated with the electricity, $\dot{C}_1$ and $\dot{C}_2$ are the cost rates associated with the entering and exiting steam, respectively, and $\dot{Z}_t$ accounts for the cost rate associated with owning and operating the turbine. With exergy costing, each of the cost rates $\dot{C}_e$, $\dot{C}_1$, and $\dot{C}_2$ is evaluated in terms of the associated rate of exergy transfer and a unit cost. Equation 7.33 then appears as

$$c_e \dot{W}_e + c_2 \dot{E}_{f2} = c_1 \dot{E}_{f1} + \dot{Z}_t \quad \text{(7.34a)}$$

The unit cost c_1 in Eq. 7.34a is given by Eq. 7.32b. In the present discussion, the same unit cost is assigned to the low-pressure steam; that is, $c_2 = c_1$. This is done on the basis that the purpose of the turbine is to generate electricity, and thus all costs associated with owning and operating the turbine should be charged to the power generated. We can regard this decision as a part of the *cost accounting* considerations that accompany the thermoeconomic analysis of thermal systems. With $c_2 = c_1$, Eq. 7.34a becomes

$$c_e \dot{W}_e = c_1(\dot{E}_{f1} - \dot{E}_{f2}) + \dot{Z}_t \quad \text{(7.34b)}$$

The first term on the right side accounts for the cost of the exergy used and the second term accounts for the cost of owning and operating the system.

Solving Eq. 7.34b for c_e, and introducing the exergetic turbine efficiency ε from Eq. 7.24

$$c_e = \frac{c_1}{\varepsilon} + \frac{\dot{Z}_t}{\dot{W}_e} \quad \text{(7.34c)}$$

This equation shows that the unit cost of the electricity is determined by the cost of the high-pressure steam and the cost of owning and operating the turbine. Because of exergy destruction within the turbine, the exergetic efficiency is invariably less than 1, and therefore the unit cost of electricity is invariably greater than the unit cost of the high-pressure steam.

Summary

By applying cost rate balances to the boiler and the turbine, we are able to determine the cost of each product of the cogeneration system. The unit cost of the electricity is determined by Eq. 7.34c and the unit cost of the low-pressure steam is determined by the expression $c_2 = c_1$ together with Eq. 7.32b. The example to follow provides a detailed illustration. The same general approach is applicable for costing the products of a wide-ranging class of thermal systems.[1]

[1]See A. Bejan, G. Tsatsaronis, and M. J. Moran, *Thermal Design and Optimization*, John Wiley & Sons, New York, 1996.

EXAMPLE 7.10

Exergy Costing of a Cogeneration System

A cogeneration system consists of a natural gas–fueled boiler and a steam turbine that develops power and provides steam for an industrial process. At steady state, fuel enters the boiler with an exergy rate of 100 MW. Steam exits the boiler at 50 bar, 466°C with an exergy rate of 35 MW. Steam exits the turbine at 5 bar, 205°C and a mass flow rate of 26.15 kg/s. The unit cost of the fuel is 1.44 cents per kW · h of exergy. The costs of owning and operating the boiler and turbine are, respectively, \$1080/h and \$92/h. The feedwater and combustion air enter with negligible exergy and cost. Expenses related to proper disposal of the combustion products are included with the cost of owning and operating the boiler. Heat transfer with the surroundings and the effects of motion and gravity are negligible. Let $T_0 = 298$ K.

(a) For the turbine, determine the power and the rate exergy exits with the steam, each in MW.

(b) Determine the unit costs of the steam exiting the boiler, the steam exiting the turbine, and the power, each in cents per kW · h of exergy.

(c) Determine the cost rates of the steam exiting the turbine and the power, each in \$/h.

SOLUTION

Known: Steady-state operating data are known for a cogeneration system that produces both electricity and low-pressure steam for an industrial process.

Find: For the turbine, determine the power and the rate exergy exits with the steam. Determine the unit costs of the steam exiting the boiler, the steam exiting the turbine, and the power developed. Also determine the cost rates of the low-pressure steam and power.

Schematic and Given Data:

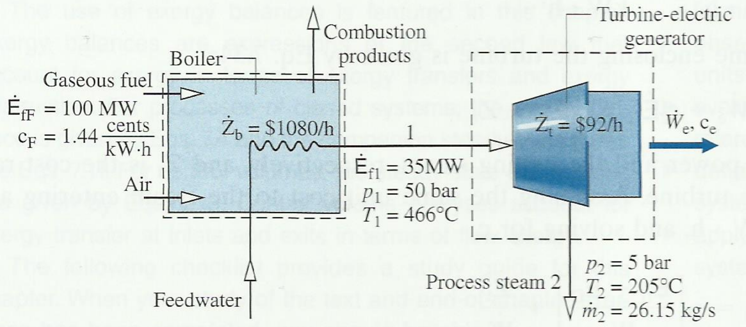

Fig. E7.10

Engineering Model:

1. Each control volume shown in the accompanying figure is at steady state.
2. For each control volume, $\dot{Q}_{cv} = 0$ and the effects of motion and gravity are negligible.
3. The feedwater and combustion air enter the boiler with negligible exergy and cost.
4. Expenses related to proper disposal of the combustion products are included with the cost of owning and operating the boiler.
5. The unit costs based on exergy of the high- and low-pressure steam are equal: $c_1 = c_2$.
6. For the environment, $T_0 = 298$ K.

Analysis:

(a) With assumption 2, the mass and energy rate balances for a control volume enclosing the turbine reduce at steady state to give

$$\dot{W}_e = \dot{m}(h_1 - h_2)$$

From Table A-4, $h_1 = 3353.54$ kJ/kg and $h_2 = 2865.96$ kJ/kg. Thus

$$\dot{W}_e = \left(26.15\frac{\text{kg}}{\text{s}}\right)(3353.54 - 2865.96)\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\ \text{MW}}{10^3\ \text{kJ/s}}\right|$$
$$= 12.75\ \text{MW}$$

Using Eq. 7.18, the difference in the rates at which exergy enters and exits the turbine with the steam is

$$\dot{E}_{f2} - \dot{E}_{f1} = \dot{m}(e_{f2} - e_{f1})$$
$$= \dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]$$

Solving for $\dot{E}_{f2}$

$$\dot{E}_{f2} = \dot{E}_{f1} + \dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]$$

With known values for $\dot{E}_{f1}$ and $\dot{m}$, and data from Table A-4, $s_1 = 6.8773$ kJ/kg · K and $s_2 = 7.0806$ kJ/kg · K, the rate exergy exits with the steam is

$$\dot{E}_{f2} = 35\ \text{MW} + \left(26.15\frac{\text{kg}}{\text{s}}\right)\left[(2865.96 - 3353.54)\frac{\text{kJ}}{\text{kg}} - 298\ \text{K}(7.0806 - 6.8773)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\ \text{MW}}{10^3\ \text{kJ/s}}\right|$$
$$= 20.67\ \text{MW}$$

(b) For a control volume enclosing the boiler, the cost rate balance reduces with assumptions 3 and 4 to give

$$c_1\dot{E}_{f1} = c_F\dot{E}_{fF} + \dot{Z}_b$$

where $\dot{E}_{fF}$ is the exergy rate of the entering fuel, c_F and c_1 are the unit costs of the fuel and exiting steam, respectively, and $\dot{Z}_b$ is the cost rate associated with owning and operating the boiler. Solving for c_1, we get Eq. 7.32b; then, inserting known values, c_1 is determined:

$$c_1 = c_F\left(\frac{\dot{E}_{fF}}{\dot{E}_{f1}}\right) + \frac{\dot{Z}_b}{\dot{E}_{f1}}$$

$$= \left(1.44\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left(\frac{100\text{ MW}}{35\text{ MW}}\right) + \left(\frac{1080\ \$/\text{h}}{35\text{ MW}}\right)\left|\frac{1\text{ MW}}{10^3\text{ kW}}\right|\left|\frac{100\text{ cents}}{1\$}\right|$$

$$= (4.11 + 3.09)\frac{\text{cents}}{\text{kW}\cdot\text{h}} = 7.2\frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

The cost rate balance for the control volume enclosing the turbine is given by Eq. 7.34a

$$c_e\dot{W}_e + c_2\dot{E}_{f2} = c_1\dot{E}_{f1} + \dot{Z}_t$$

where c_e and c_2 are the unit costs of the power and the exiting steam, respectively, and $\dot{Z}_t$ is the cost rate associated with owning and operating the turbine. Assigning the same unit cost to the steam entering and (1) exiting the turbine, $c_2 = c_1 = 7.2$ cents/kW · h, and solving for c_e

$$c_e = c_1\left[\frac{\dot{E}_{f1} - \dot{E}_{f2}}{\dot{W}_e}\right] + \frac{\dot{Z}_t}{\dot{W}_e}$$

Inserting known values

(2)

$$c_e = \left(7.2\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left[\frac{(35 - 20.67)\text{ MW}}{12.75\text{ MW}}\right] + \left(\frac{92\$/\text{h}}{12.75\text{ MW}}\right)\left|\frac{1\text{ MW}}{10^3\text{ kW}}\right|\left|\frac{100\text{ cents}}{1\$}\right|$$

$$= (8.09 + 0.72)\frac{\text{cents}}{\text{kW}\cdot\text{h}} = 8.81\frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

(c) For the low-pressure steam and power, the cost rates are, respectively,

(3)

$$\dot{C}_2 = c_2\dot{E}_{f2}$$

$$= \left(7.2\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(20.67\text{ MW})\left|\frac{10^3\text{ kW}}{1\text{ MW}}\right|\left|\frac{\$1}{100\text{ cents}}\right|$$

$$= \$1488/\text{h}$$

$$\dot{C}_e = c_e\dot{W}_e$$

$$= \left(8.81\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(12.75\text{ MW})\left|\frac{10^3\text{ kW}}{1\text{ MW}}\right|\left|\frac{\$1}{100\text{ cents}}\right|$$

$$= \$1123/\text{h}$$

---

(1) The purpose of the turbine is to generate power, and thus all costs associated with owning and operating the turbine are charged to the power generated.

(2) Observe that the unit costs c_1 and c_e are significantly greater than the unit cost of the fuel.

(3) Although the unit cost of the steam is less than the unit cost of the power, the steam *cost* rate is greater because the associated exergy rate is much greater.

✓ Skills Developed

Ability to...

- ❑ evaluate exergy quantities required for exergy costing.
- ❑ apply exergy costing.

QuickQUIZ If the unit cost of the fuel were to double to 2.88 cents/kW · h, what would be the change in the unit cost of power, expressed as a percentage, keeping all other given data the same? Ans. +53%.