

Chapter 7

Exergy Analysis

Learning Outcomes

- ▶ Demonstrate understanding of key concepts related to exergy analysis, including the exergy reference environment, the dead state, exergy transfer, and exergy destruction.
- ▶ Evaluate exergy at a state and exergy change between two states, using appropriate property data.

Learning Outcomes, cont.

- ▶ Apply exergy balances to closed systems and to control volumes at steady state.
- ▶ Define and evaluate exergetic efficiencies.
- ▶ Apply exergy costing to heat loss and simple cogeneration systems.

Exergy Analysis

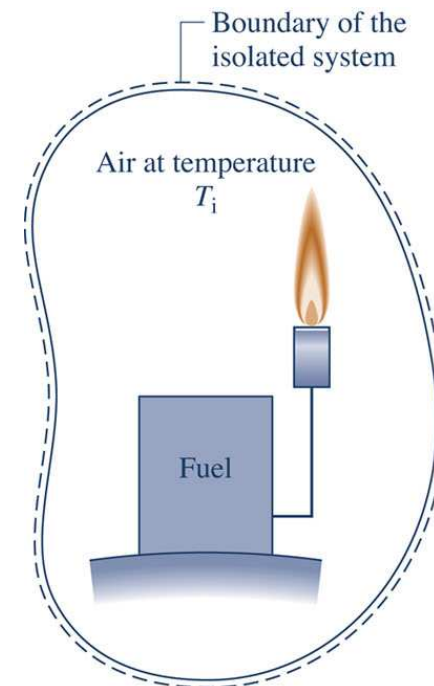
▶ **Exergy analysis** contributes to the goal of making more effective use of nonrenewable energy resources: natural gas, coal, and oil, by determining the locations, types, and true magnitudes of waste and loss in systems fueled by such resources.

▶ **Exergy analysis** is also relevant for designing more effective thermal systems of all types, guiding efforts to reduce inefficiencies in such systems, and evaluating system economics.

Introducing Exergy with a Thought Experiment

▶ When you fill an automobile's fuel tank with gasoline, it is the **exergy of the gasoline** you seek and for which you pay.

▶ Exergy is not just another aspect of energy. **Exergy and energy are related but distinctly different quantities.** These differences are explored with the figure at right, which shows an isolated system consisting initially of a small container of fuel surrounded by air in abundance.

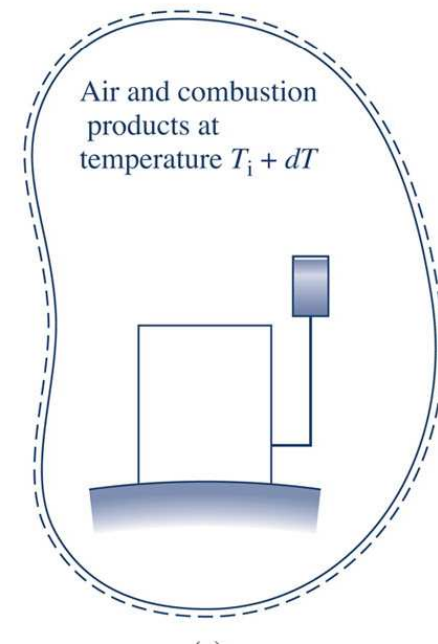


Introducing Exergy with a Thought Experiment

▶ Suppose the fuel burns so **finally** there is a **slightly warm mixture of air and the combustion products** formed.

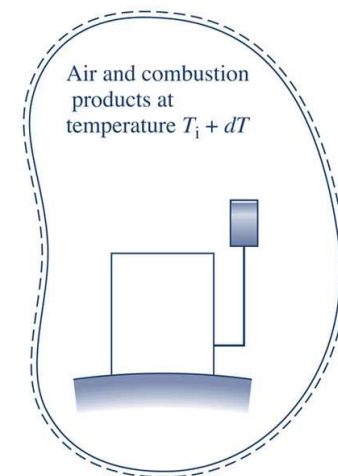
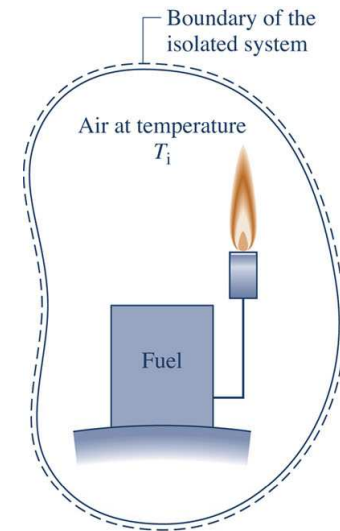
▶ Since air is abundantly present, the **temperature of the final mixture is nearly the same as the initial air temperature.**

▶ The total **quantity** of energy associated with the system is **constant** because no energy transfers take place across the boundary of an isolated system and, by the first law of thermodynamics, energy is conserved.



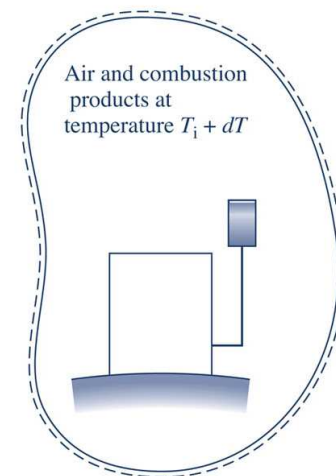
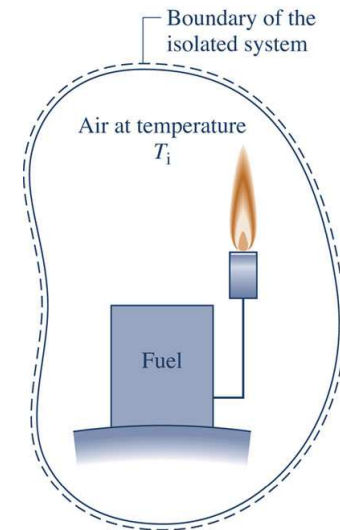
Introducing Exergy with a Thought Experiment

- ▶ The initial fuel-air combination has a much greater *potential for use* than the final warm mixture. For instance, the fuel might be used to generate electricity, produce steam, or power a car whereas the final warm mixture is clearly unsuited for such applications.
- ▶ Actually, during the process shown in the figures the *initial potential for use* is predominately *destroyed* owing to the irreversible nature of that process.



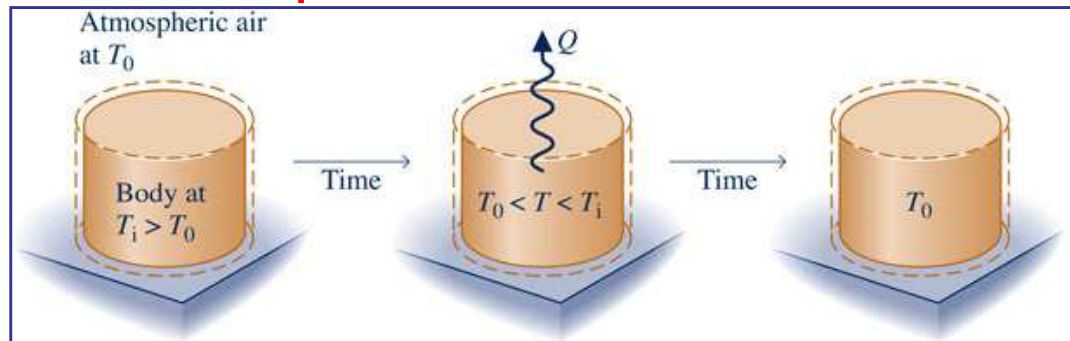
Introducing Exergy with a Thought Experiment

- ▶ The fuel present initially also has economic value, but **economic value diminishes as fuel is consumed**. The final warm mixture has negligible economic value.
- ▶ **Exergy** is the **property** that quantifies the **potential for use** and it is exergy that **has economic value**.

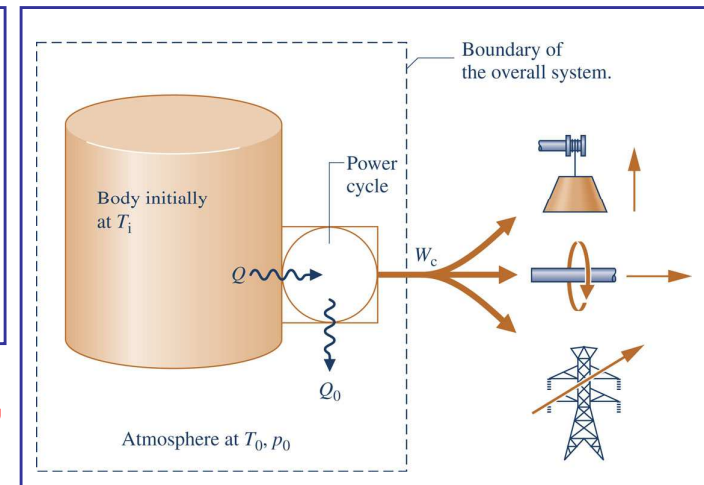


Conceptualizing Exergy

► Consider a body at temperature T_i placed in contact with the atmosphere at temperature T_0 . If $T_i > T_0$, the body cools spontaneously until it is in thermal equilibrium with the atmosphere.



► However, by controlling the cooling, work can be developed as shown.



► Instead of the body cooling spontaneously, heat transfer Q passes to a power cycle that develops work W_c . The work is fully available for lifting a weight, developing shaft work, or generating electricity.

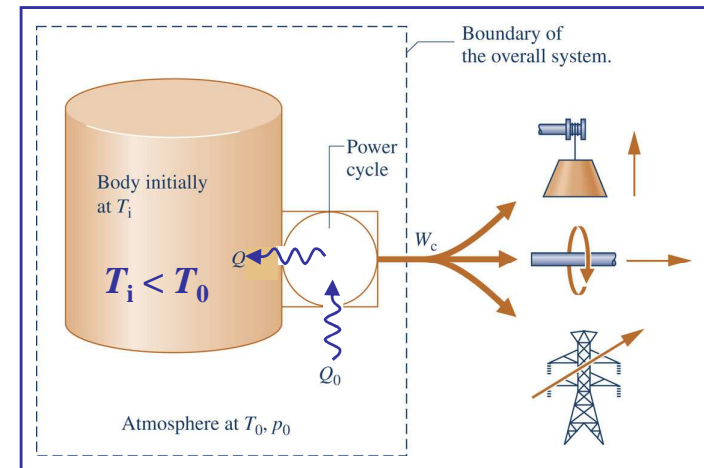
Conceptualizing Exergy

- ▶ Since the power cycle undergoes no net change in state, the **potential for developing work W_c arises solely because the initial state of the body differs from that of the atmosphere** – namely, $T_i > T_0$.
- ▶ The body eventually achieves thermal equilibrium with the atmosphere. **At equilibrium, the body and atmosphere each possess energy, but there is no longer any potential for developing work from the two** because no further interaction can occur between them.
- ▶ **Exergy is the maximum theoretical value for the work W_c .** Maximum work is realized only when there are no irreversibilities as the body cools to equilibrium with the atmosphere.

Conceptualizing Exergy

► In **spontaneous cooling to T_0** , no work is obtained and so the initial potential to develop work, exergy, is **destroyed**.

► Work W_c also can be developed when $T_i < T_0$. In that case, the heat transfers shown on the figure reverse direction and work is developed as the body warms to equilibrium with the atmosphere.



► As before, **the potential for developing work arises solely because the initial state of the body differs from that of the atmosphere** – namely $T_i < T_0$.

Defining Exergy

- ▶ The term *exergy reference environment* – or simply the *environment* – refers to a model for the Earth's atmosphere: a simple compressible system that is large in extent and uniform in pressure, p_0 , and temperature, T_0 . The values of p_0 and T_0 are normally taken as typical ambient values, such as 1 bar and 25°C.
- ▶ Exergy is the maximum theoretical work obtainable from an *overall* system consisting of a system and the environment as the system comes into equilibrium with the environment (passes to the dead state).
- ▶ The term *dead state* refers to when a system of interest is at T_0 and p_0 and at *rest* relative to the environment. At the dead state there can be no interaction between system and environment, and thus there is no potential for developing work.

Defining Exergy

► Using energy and entropy balances, the following **expression** is obtained **for the exergy, E, of a system at a specified state,**

$$E = (U - U_0) + p_0(V - V_0) - T_0(S - S_0) + \text{KE} + \text{PE} \quad (\text{Eq. 7.1})$$

where U , V , S , KE, and PE denote, respectively, internal energy, volume, entropy, kinetic energy, and potential energy of the system at the specified state. U_0 , V_0 , and S_0 denote internal energy, volume, and entropy, respectively, of the system when at the dead state. At the dead state, the kinetic and potential energy of the system are each zero.

► Once the environment is specified, a value can be assigned to exergy in terms of property values for the system only, so exergy can be regarded as a property of the system. **Exergy is an extensive property.**

Defining Exergy

► Expressing **Eq. 7.1** on a unit mass basis, the **specific exergy** is

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + V^2/2 + gz \quad \text{(Eq. 7.2)}$$

► Using **Eq. 7.1**, the **change in exergy between two states** is

$$E_2 - E_1 = (U_2 - U_1) + p_0(V_2 - V_1) - T_0(S_2 - S_1) + (KE_2 - KE_1) + (PE_2 - PE_1)$$

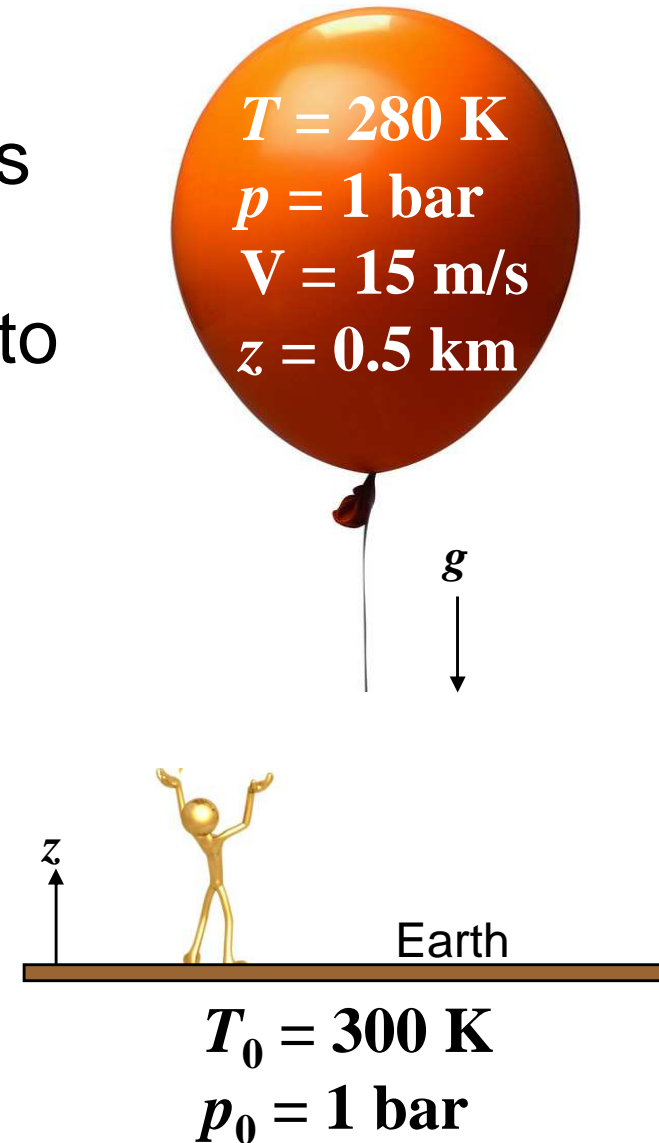
(Eq. 7.3)

Defining Exergy

- ▶ If temperature and/or pressure of a system differ from that of the environment, the system has exergy. More precisely, the system has a ***thermomechanical*** contribution to its exergy. Another contribution, called ***chemical exergy***, arises when there is a chemical composition difference between the system and environment.
- ▶ The thermomechanical contribution to exergy suffices for the applications of Chapter 7. Still, **for gasoline and other fuels chemical exergy is the most significant aspect**. Chemical exergy is considered in Chapter 13.

Calculating Exergy

Example: A balloon filled with oxygen (O_2) at **280 K**, **1 bar** moves with a velocity of **15 m/s** at an elevation of **0.5 km**, each relative to Earth's surface where $T_0 = 300$ K, $p_0 = 1$ bar. Using the ideal gas model, determine the specific exergy of the oxygen, in kJ/kg. Take $g = 9.807$ m/s².



Calculating Exergy



Solution: The specific exergy is given by

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + V^2/2 + gz$$

► Using the ideal gas model and data from **Table A-23**, as needed

$$(u - u_0) = \frac{\bar{u} - \bar{u}_0}{M} = \frac{(5822 - 6242) \text{ kJ/kmol}}{32 \text{ kg/kmol}} = -13.13 \text{ kJ/kg}$$

$$p_0(v - v_0) = p_0 \left(\frac{RT}{p} - \frac{RT_0}{p_0} \right). \text{ Since } p = p_0,$$

$$p_0(v - v_0) = R(T - T_0) = \left(\frac{8.314}{32} \right) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (280 - 300) \text{ K} = -5.20 \text{ kJ/kg}$$

Calculating Exergy



$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + V^2/2 + gz$$

$$-T_0(s - s_0) = -T_0 \left(s^o - s_0^o - R \ln \frac{p}{p_0} \right). \text{ Since } p = p_0,$$

$$-T_0(s - s_0) = -T_0 \left(\frac{\bar{s}^o - \bar{s}_0^o}{M} \right)$$

$$-T_0(s - s_0) = -(300 \text{ K}) \frac{(203.191 - 205.213) \text{ kJ/kmol} \cdot \text{K}}{32 \text{ kg/kmol}} = +18.96 \text{ kJ/kg}$$

Calculating Exergy



$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + V^2/2 + gz$$

$$V^2 / 2 + gz = \left((15 \text{ m/s})^2 / 2 + (9.807 \text{ m/s}^2)(500 \text{ m}) \right) \left[\frac{1 \text{ N}}{1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}} \right] \left[\frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right]$$
$$V^2 / 2 + gz = +5.02 \text{ kJ/kg}$$

► Collecting results,

$$e = (-13.13 - 5.20 + 18.96 + 5.02) \text{ kJ/kg} = \mathbf{5.65 \text{ kJ/kg}}$$

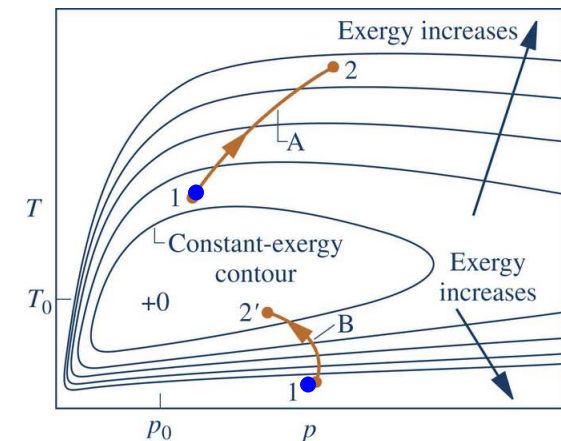
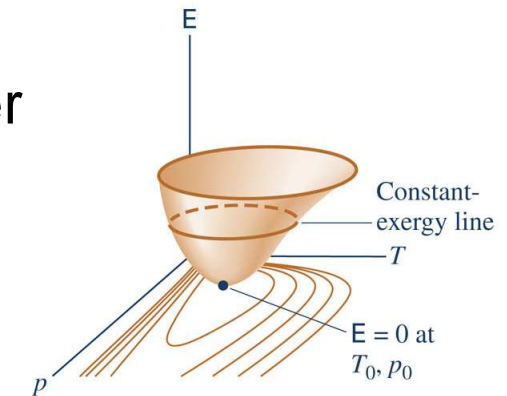
Illustrating Exergy Change

► The figure at the right shows an **exergy-temperature-pressure surface** for a gas together with **constant-exergy contours projected on the temperature-pressure coordinates**.

► For a system undergoing **Process A**, exergy increases as its state moves **away** from the dead state: from **1** to **2**. That is, **exergy change is positive**.

► In **Process B**, exergy decreases as the state moves **toward** the dead state: from **1'** to **2'**. That is, **exergy change is negative**.

► The figure also shows that the **value of exergy is positive at states other than the dead state, is zero at the dead state, and is never negative**.



Introducing the Closed System Exergy Balance

- ▶ Mass, energy, entropy, and exergy are all extensive properties.
- ▶ Just as mass, energy, and entropy are accounted for by mass, energy, and entropy balances, **exergy is accounted for by an exergy balance.**
- ▶ Like mass, energy, and entropy, **exergy can be transferred across the system boundary.**
- ▶ Mass and energy are conserved quantities but **entropy and exergy are not conserved.**

Introducing the Closed System Exergy Balance

► The **exergy balance for a closed system** is developed by combining the closed system energy and entropy balances. The result is

$$\frac{E_2 - E_1}{\text{exergy change}} = \frac{\int_1^2 \left(1 - \frac{T_0}{T_b}\right) \delta Q - [W - p_0(V_2 - V_1)]}{\text{exergy transfers}} - \frac{T_0 \sigma}{\text{exergy destruction}}$$

(Eq. 7.4a)

Introducing the Closed System Exergy Balance

► The **first two terms** on the right side account for exergy transfer accompanying heat transfer and work, respectively:

$$E_q = \left[\begin{array}{c} \text{exergy transfer} \\ \text{accompanying heat} \\ \text{transfer} \end{array} \right] = \int_1^2 \left(1 - \frac{T_0}{T_b} \right) \delta Q \quad \text{(Eq. 7.5)}$$

where T_b is the temperature on the boundary where heat transfer occurs,

$$E_w = \left[\begin{array}{c} \text{exergy transfer} \\ \text{accompanying work} \end{array} \right] = [W - p_0(V_2 - V_1)] \quad \text{(Eq. 7.6)}$$

Introducing the Closed System Exergy Balance

► The **third term** on the right accounts for the destruction of exergy due to irreversibilities within the system:

$$E_d = T_0\sigma \quad (\text{Eq. 7.7})$$

In accord with the interpretation of the entropy production term of the entropy balance

$$E_d: \begin{cases} = 0 & \text{(no irreversibilities present within the system)} \\ > 0 & \text{(irreversibilities present within the system)} \\ < 0 & \text{(impossible)} \end{cases}$$

Introducing the Closed System Exergy Balance

- ▶ There are two ways for evaluating the exergy destruction:
 - 1) Solve the exergy balance for exergy destruction when all other terms are known.
 - 2) Apply **Eq. 7.7** when the entropy production is known from an entropy balance.

Exergy Rate Balance for Closed Systems

- ▶ On a time rate basis, the closed system exergy rate balance is:

$$\frac{dE}{dt} = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \left(\dot{W} - p_0 \frac{dV}{dt}\right) - \dot{E}_d \quad (\text{Eq. 7.10})$$

- ▶ At steady state, **Eq. 7.10** reduces to give

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W} - \dot{E}_d \quad (\text{Eq. 7.11a})$$

Exergy Rate Balance for Closed Systems

$$\frac{dE}{dt} = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \left(\dot{W} - p_0 \frac{dV}{dt}\right) - \dot{E}_d$$

(Eq. 7.10)

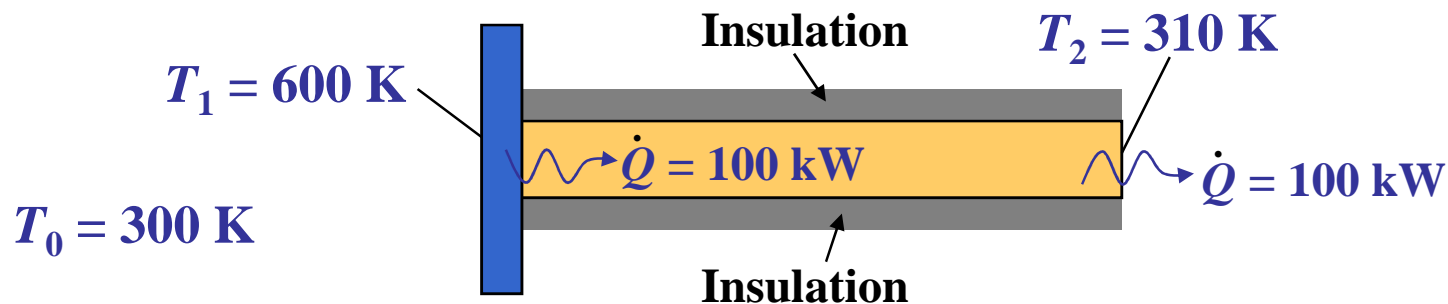
$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W} - \dot{E}_d$$

(Eq. 7.11a)

- In both expressions, the term $\dot{E}_{qj} = \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j$ represents the **time rate of exergy transfer accompanying heat transfer** at the rate \dot{Q}_j occurring where the temperature on the boundary is T_j .
- Also, in both expressions \dot{E}_d accounts for the **time rate of exergy destruction due to irreversibilities** within the system.
- Finally, note that for a system at steady state, the **rate of exergy transfer accompanying the power \dot{W}** is simply the **power**.

Exergy Rate Balance for Closed Systems

Example: A cylindrical rod insulated on its lateral surface is in contact at one end with a wall at $T_1 = 600 \text{ K}$. The other end, at $T_2 = 310 \text{ K}$, is exposed to the atmosphere, which is at $T_0 = 300 \text{ K}$. At steady state, energy is conducted by heat transfer through the rod at a rate of 100 kW . Determine, in kW, (a) the rates of exergy transfer accompanying heat transfer into and out of the rod, (b) the rate of exergy destruction within the rod, and (c) an accounting of the exergy entering the rod.



Exergy Rate Balance for Closed Systems

(a) The rates of exergy transfer accompanying heat transfer into and out of the rod are, respectively

$$\dot{E}_{q1} = \left(1 - \frac{T_0}{T_1}\right) \dot{Q} = \left(1 - \frac{300 \text{ K}}{600 \text{ K}}\right) (100 \text{ kW}) = 50 \text{ kW}$$

$$\dot{E}_{q2} = \left(1 - \frac{T_0}{T_2}\right) \dot{Q} = \left(1 - \frac{300 \text{ K}}{310 \text{ K}}\right) (100 \text{ kW}) = 3 \text{ kW}$$

(b) Applying the exergy rate balance, **Eq. 7.11a**

$$0 = \dot{E}_{q1} - \dot{E}_{q2} - \cancel{\dot{W}} - \dot{E}_d$$

$$\dot{E}_d = \dot{E}_{q1} - \dot{E}_{q2} = 50 \text{ kW} - 3 \text{ kW} = 47 \text{ kW}$$

The irreversibility in this case is spontaneous heat transfer within the rod from the high-temperature end to the low-temperature end.

Exergy Rate Balance for Closed Systems

(c) While the rates of heat transfer are the same at each end of the rod, the rates of exergy transfer at the ends are much different. The exergy transfer rates provide truer measures of thermodynamic value than the heat transfer rate. The thermodynamic value of a heat transfer depends on the temperature at which it occurs.

- ▶ The magnitude of the exergy transfer rate at the high-temperature end signals an opportunity for doing something useful.
- ▶ The smaller exergy transfer rate at the low-temperature end indicates much reduced usefulness, but still an opportunity.

Exergy Rate Balance for Closed Systems

- ▶ An **exergy accounting** for the rod reads:

Rate of exergy in:	50 kW
Disposition of the Exergy:	
▶ Rate of exergy out:	3 kW (6%)
▶ Rate of exergy destruction:	47 kW (94%)

▶ In principle the exergy transferred from the rod can be used for some purpose. Accordingly, it is accounted for here as exergy transfer out of the rod. However, as this exergy is not used in the surroundings, it is more appropriately charged as an **exergy loss** for the rod.

▶ The **exergy transferred from the rod is fully destroyed in the surroundings owing to heat transfer irreversibility there.** In the end, all of the exergy entering the rod is destroyed, thereby missing a seeming opportunity.

Exergy Rate Balance for Control Volumes at Steady State

- ▶ Like mass, energy, and entropy, **exergy can be transferred into and out of a control volume where mass enters and exits.**
- ▶ Since this is the principal difference between the closed system and control volume exergy rate balances, the **steady-state form for control volumes** can be obtained by modifying the closed system form to account for such exergy transfer. The result is

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W}_{cv} + \sum_i \dot{m}_i e_{fi} - \sum_e \dot{m}_e e_{fe} - \dot{E}_d \quad \text{(Eq. 7.13a)}$$

Exergy Rate Balance for Control Volumes at Steady State

► In **Eq. 7.13a**, e_{fi} accounts for the exergy per unit of mass entering at inlet i and e_{fe} accounts for the exergy per unit mass exiting at exit e . These terms, known as the **specific flow exergy**, are derived using energy and entropy balances, and take the form

$$e_f = h - h_0 - T_0(s - s_0) + \frac{V^2}{2} + gz \quad \text{(Eq. 7.14)}$$

where h and s represent the **specific enthalpy and entropy**, respectively, **at the inlet or exit** under consideration; h_0 and s_0 are the values of these properties evaluated at T_0, p_0 .

Exergy Rate Balance for Control Volumes at Steady State

► **Eq. 7.13a** can be expressed more compactly as

$$0 = \sum_j \dot{E}_{qj} - \dot{W}_{cv} + \sum_i \dot{E}_{fi} - \sum_e \dot{E}_{fe} - \dot{E}_d \quad \text{(Eq. 7.13b)}$$

where

$$\dot{E}_{qj} = \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j \quad \text{(Eq. 7.15)}$$

$$\dot{E}_{fi} = \dot{m}_i e_{fi} \quad \text{(Eq. 7.16a)}$$

$$\dot{E}_{fe} = \dot{m}_e e_{fe} \quad \text{(Eq. 7.16b)}$$

Exergy Rate Balance for Control Volumes at Steady State

► For a **one-inlet, one-exit control volume at steady state**, **Eq. 7.13a** reduces to

$$0 = \sum_j \left(1 - \frac{T_0}{T_j} \right) \dot{Q}_j - \dot{W}_{cv} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d \quad \text{(Eq. 7.17)}$$

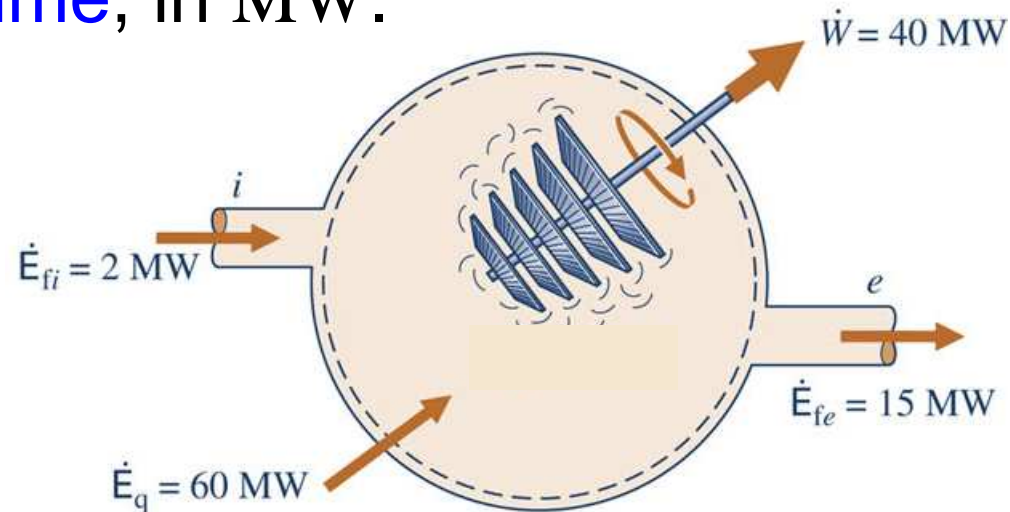
where 1 and 2 denote the inlet and exit, respectively, and \dot{m} is the common mass flow rate at these locations. The term $(e_{f1} - e_{f2})$ is evaluated using **Eq. 7.14** as

$$e_{f1} - e_{f2} = (h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

(Eq. 7.18)

Exergy Rate Balance for Control Volumes at Steady State

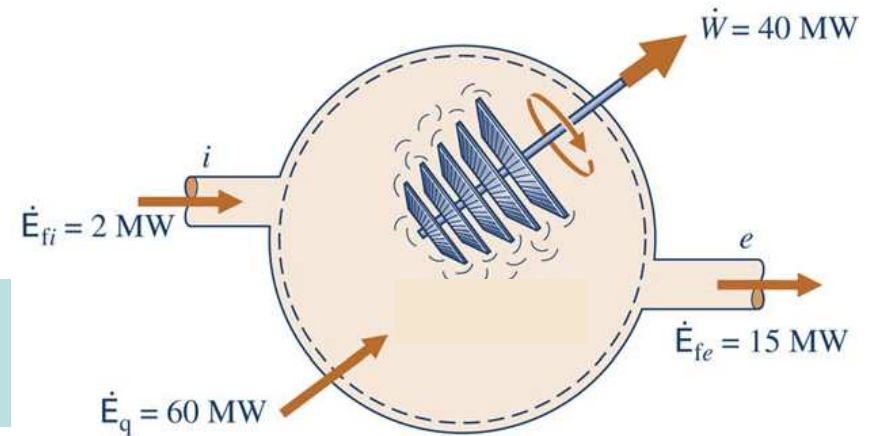
Example: The figure shows a control volume at steady state labeled with exergy transfer rates accompanying shaft work, accompanying heat transfer, and where mass enters and exits. Applying the control volume form of the exergy rate balance, determine the rate of exergy destruction within the control volume, in MW.



Exergy Rate Balance for Control Volumes at Steady State

► In this application, **Eq. 7.13b** takes the form

$$0 = \dot{E}_q - \dot{W}_{cv} + \dot{E}_{fi} - \dot{E}_{fe} - \dot{E}_d$$



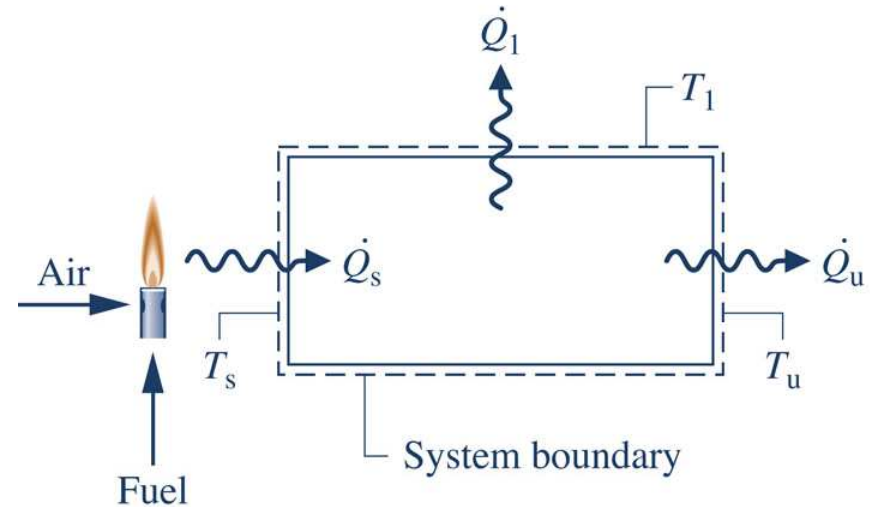
► Solving for \dot{E}_d and substituting values from the figure

$$\dot{E}_d = 60 \text{ MW} - 40 \text{ MW} + 2 \text{ MW} - 15 \text{ MW} = \mathbf{7 \text{ MW}}$$

Exergetic Efficiency

▶ Exergy-based efficiencies developed using exergy balances are called **exergetic efficiencies**.

▶ To distinguish exergy-based and energy-based efficiencies, consider the system shown in the figure. **The system represents a range of applications where fuel is consumed to provide heating.**



▶ All energy transfers shown are in the direction of the arrows.

▶ The system receives energy by heat transfer at the rate \dot{Q}_s at the source temperature T_s and delivers \dot{Q}_u at the use temperature T_u .

▶ Energy is lost by heat transfer at the rate \dot{Q}_l at temperature T_1 .

▶ There is no work and operation is at steady state.

Exergetic Efficiency

- ▶ Applying closed system energy and exergy rate balances at steady-state

$$\frac{dE}{dt} = (\dot{Q}_s - \dot{Q}_u - \dot{Q}_l) - \dot{W}^0$$

$$\frac{dE^0}{dt} = \left[\left(1 - \frac{T_0}{T_s}\right)\dot{Q}_s - \left(1 - \frac{T_0}{T_u}\right)\dot{Q}_u - \left(1 - \frac{T_0}{T_l}\right)\dot{Q}_l \right] - \left[\dot{W}^0 - p_0 \frac{dV^0}{dt} \right] - \dot{E}_d$$

- ▶ These equations can be rewritten as

$$\dot{Q}_s = \dot{Q}_u + \dot{Q}_l \quad \text{(Eq. 7.19a)}$$

$$\left(1 - \frac{T_0}{T_s}\right)\dot{Q}_s = \left(1 - \frac{T_0}{T_u}\right)\dot{Q}_u + \left(1 - \frac{T_0}{T_l}\right)\dot{Q}_l + \dot{E}_d \quad \text{(Eq. 7.19b)}$$

Exergetic Efficiency

► **Equation 7.19a** shows that the energy carried in by heat transfer, \dot{Q}_s , is either used, \dot{Q}_u , or lost, \dot{Q}_l . This can be described by an energy-based efficiency in the form product/input as

$$\eta = \frac{\dot{Q}_u}{\dot{Q}_s} \quad (\text{Eq. 7.20})$$

The value of η can be increased by applying insulation to reduce the loss. In the limit as the loss is eliminated, the value of η approaches 1 (100%).

Exergetic Efficiency

► **Equation 7.19b** shows that the **exergy** carried in accompanying heat transfer \dot{Q}_s is either transferred from the system accompanying the heat transfers \dot{Q}_u and \dot{Q}_l or destroyed by irreversibilities within the **system**. This can be described by an exergy-based efficiency in the form product/input as

$$\varepsilon = \frac{(1 - T_0/T_u)\dot{Q}_u}{(1 - T_0/T_s)\dot{Q}_s} \quad \text{(Eq. 7.21a)}$$

Exergetic Efficiency

▶ Introducing **Eq. 7.20** gives

$$\varepsilon = \eta \left(\frac{1 - T_0/T_u}{1 - T_0/T_s} \right) \quad \text{(Eq. 7.21b)}$$

▶ Study of this expression shows that **there are two ways to increase the value of the exergetic efficiency**:

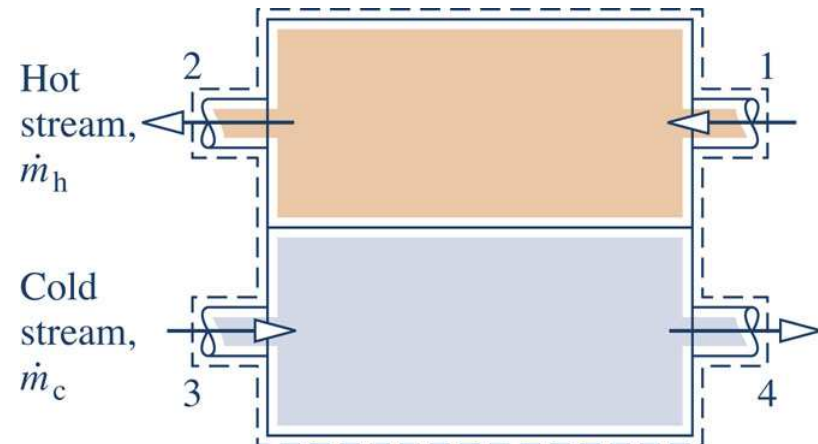
- ▶ Increase the value of η to as close to unity as practical.
- ▶ Increase the use temperature, T_u , so it better matches the source temperature, T_s .

Still, **the limit of 100% exergetic efficiency is not a practical objective**. In most applications where fuel is consumed to provide heating, ε is much less than 100% and is less than 10% in domestic water heaters. In such cases, typically there is not a good match between source and use temperature.

Exergetic Efficiency

Example: Steady-state operating data are provided in the table for the counterflow heat exchanger shown in the figure. The **hot stream** is waste water from an industrial processing unit. The **cold stream** is a gas being preheated for some application. Heat transfer from the overall control volume and kinetic and potential energy effects can be ignored. The ratio of the mass flow rates is $\dot{m}_c/\dot{m}_h = 1/3$. For the heat exchanger, devise and evaluate **(a)** an energy-based efficiency and **(b)** an exergetic efficiency.

State	h (kJ/kg)	e_f (kJ/kg)
1	340	24.4
2	300	16.0
3	500	36.8
4	620	47.0



Exergetic Efficiency

(a) Reducing an energy rate balance for the heat exchanger

$$0 = \dot{m}_h(h_1 - h_2) + \dot{m}_c(h_3 - h_4)$$

► This can be rearranged to read

$$\dot{m}_h(h_1 - h_2) = \dot{m}_c(h_4 - h_3)$$

► The term on the **left** accounts for the decrease in energy of the hot stream. The term on the **right** accounts for the increase in energy of the cold stream. A **ratio** of these terms gives an **energy-based efficiency**:

$$\eta = \frac{\dot{m}_c(h_4 - h_3)}{\dot{m}_h(h_1 - h_2)}$$

Exergetic Efficiency

- ▶ Substituting known values

$$\eta = \frac{(1/3)(620 - 500)\text{kJ/kg}}{(340 - 300)\text{kJ/kg}} = \mathbf{1 = (100\%)}$$

The energy-based efficiency is 100% because in the absence of heat transfer from the overall control volume the decrease in hot stream energy equals the increase in cold stream energy.

Exergetic Efficiency

(b) The exergy rate balance, **Eq. 7.13a**, reduces to

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W}_{cv}^0 + (\dot{m}_h e_{f1} + \dot{m}_c e_{f3}) - (\dot{m}_h e_{f2} + \dot{m}_c e_{f4}) - \dot{E}_d$$

► This can be rearranged to read

$$\dot{m}_h (e_{f1} - e_{f2}) = \dot{m}_c (e_{f4} - e_{f3}) + \dot{E}_d$$

► The term on the **left** accounts for the decrease in the exergy of the hot stream. The first term on the **right** accounts for the increase in exergy of the cold stream. Regarding the hot stream as supplying the exergy increase of the cold stream as well as the exergy destroyed, we can write an **exergetic heat exchanger efficiency** as

$$\varepsilon = \frac{\dot{m}_c (e_{f4} - e_{f3})}{\dot{m}_h (e_{f1} - e_{f2})}$$

Exergetic Efficiency

- ▶ Substituting known values

$$\varepsilon = \frac{(1/3)(47.0 - 36.8) \text{ kJ/kg}}{(24.4 - 16.0) \text{ kJ/kg}} = \mathbf{0.4 (40\%)}$$

- ▶ Only 40% of the decrease in hot stream exergy is transferred to the cold stream. The rest is destroyed accompanying stream-to-stream heat transfer within the heat exchanger.
- ▶ An exergetic efficiency of 100% is not a practical objective for counterflow heat exchangers.

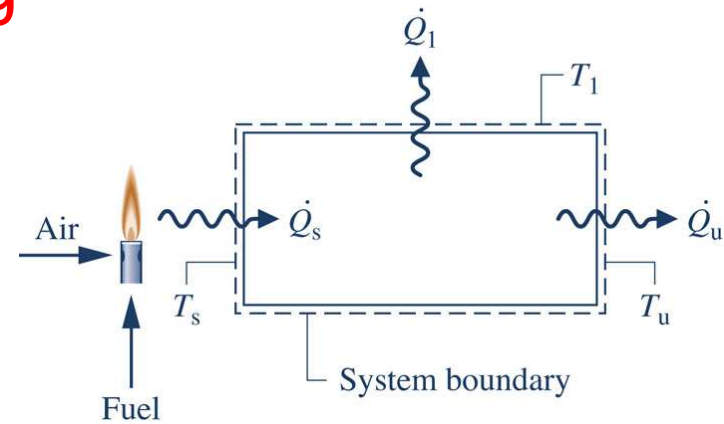
Thermoeconomics

- ▶ The term *thermoeconomics* is employed in **Chapter 7** for methodologies that combine **exergy analysis** and other aspects of engineering thermodynamics with **engineering economics** for
 - ▶ **Optimization studies** during design of new thermal systems, and
 - ▶ **Process improvements** of existing thermal systems.

Exergy Costing of Heat Loss

- ▶ An exergy loss accompanying heat transfer not only has **thermodynamic value** but also **economic value**.

Consider again the system at the right where fuel is consumed to provide heating.



- ▶ The **rate of exergy loss accompanying heat loss** at the rate \dot{Q}_1 is

$$\left(1 - \frac{T_0}{T_1}\right) \dot{Q}_1$$

Exergy Costing of Heat Loss

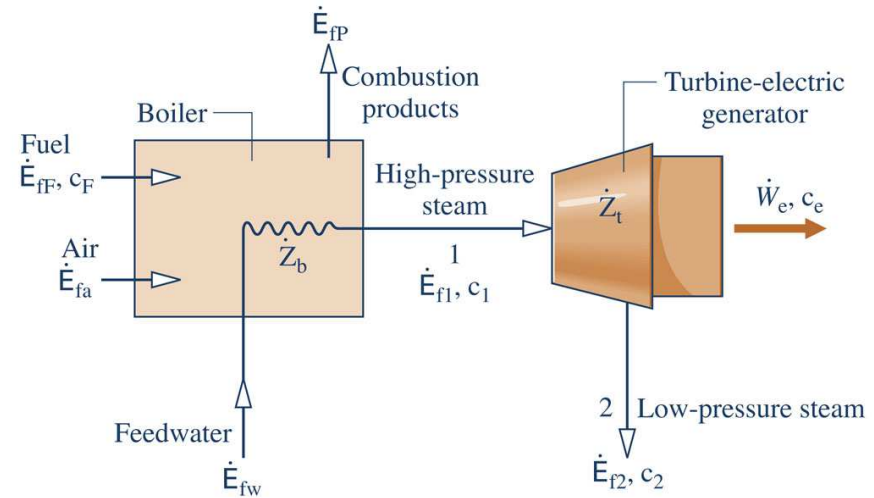
- ▶ Since the source of the exergy lost is the exergy entering with the fuel, the **economic value of the loss can be accounted for in terms of the unit cost of the fuel based on exergy**, c_F (in \$ per kW·h of exergy, for example), as

$$\left[\begin{array}{l} \text{cost rate of heat loss} \\ \dot{Q}_1 \text{ at temperature } T_1 \end{array} \right] = c_F(1 - T_0/T_1)\dot{Q}_1 \quad \text{(Eq. 7.22)}$$

- ▶ Accordingly, the **cost of a heat loss is greater at higher temperatures than at lower temperatures**. This outcome can be invoked to guide decisions about whether or not spending money on insulation to reduce heat loss is cost-effective.

Exergy Costing of a Cogeneration System

- ▶ To illustrate further aspects of thermoeconomics, consider a turbine that receives high-pressure steam from a boiler and produces electricity and low-pressure steam for use in some process.



- ▶ Assuming the turbine operates at **steady state** with **no significant heat transfer with its surroundings** and **negligible effects of kinetic and potential energy**, the energy rate balance, **Eq. 4.20a**, reduces as follows

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m} \left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$

$$\dot{W}_e = \dot{m}(h_1 - h_2)$$

Exergy Costing of a Cogeneration System

- ▶ With the same modeling, the exergy rate balance, **Eq. 7.17**, reduces as follows

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W}_{cv} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

- ▶ This equation can be rearranged to read

$$\dot{m}(e_{f1} - e_{f2}) = \dot{W}_e + \dot{E}_d$$

- ▶ The term on the left is the decrease in flow exergy from turbine inlet to exit. The equation shows that the **flow exergy decrease is accounted for by the electric power developed and the rate exergy is destroyed.**
- ▶ The **exergetic turbine efficiency**, ε , gauges how effectively the flow exergy decrease is converted to the desired product, electric power:

$$\varepsilon = \frac{\dot{W}_e}{\dot{m}(e_{f1} - e_{f2})} \quad (7.24)$$

Exergy Costing of a Cogeneration System

- ▶ Thus far the analysis of the turbine has used the first and second laws of thermodynamics in the form of energy and exergy balances, respectively. **Thermoeconomics also requires cost accounting** as embodied in the following **cost rate balance** for the turbine

$$\dot{C}_e + \dot{C}_2 = \dot{C}_1 + \dot{Z}_t \quad \text{(Eq. 7.33)}$$

where \dot{C}_e is the **cost rate of the electricity**, \dot{C}_2 is the **cost rate of the exiting steam**, \dot{C}_1 is the **cost rate of the entering steam**, and \dot{Z}_t accounts for the **cost rate associated with owning and operating the turbine**. Each cost rate has units of \$/h, for example.

Exergy Costing of a Cogeneration System

- ▶ With **exergy costing**, \dot{C}_e , \dot{C}_1 , and \dot{C}_2 are each **evaluated in terms of the associated rate of exergy transfer and a unit cost** (in \$ per kW·h of exergy, for example). **Eq. 7.33** then appears as

$$c_e \dot{W}_e + c_2 \dot{E}_{f2} = c_1 \dot{E}_{f1} + \dot{Z}_t \quad (\text{Eq. 7.34a})$$

where $\dot{E}_{f1} = \dot{m}e_{f1}$ and $\dot{E}_{f2} = \dot{m}e_{f2}$.

- ▶ The **unit cost of the high-pressure steam**, c_1 , is determined from a thermoeconomic analysis of the boiler.
- ▶ For purposes of illustration here, **the unit cost of the low-pressure steam**, c_2 , is assumed equal to c_1 . We can regard such a decision as part of the cost accounting required in thermoeconomics.

Exergy Costing of a Cogeneration System

▶ With these considerations, **Eq. 7.34a** becomes

$$c_e \dot{W}_e = c_1 (\dot{E}_{f1} - \dot{E}_{f2}) + \dot{Z}_t \quad (\text{Eq. 7.34b})$$

▶ The first term on the right accounts for the **cost of exergy used**.

▶ The second term on the right accounts for the **cost of owning and operating the turbine**.

Exergy Costing of a Cogeneration System

- Solving **Eq. 7.34b** for the unit cost of the electricity, c_e , and introducing the exergetic turbine efficiency, **Eq. (7.24)**

$$c_e = \frac{c_1}{\varepsilon} + \frac{\dot{Z}_t}{\dot{W}_e} \quad \text{(Eq. 7.34c)}$$

This expression shows that the **unit cost of electricity is invariably greater than the unit cost of high-pressure steam** because ε is invariably less than 1 (100%) **and** there is a cost of owning and operating the turbine.

Exergy Costing of a Cogeneration System

► **Sample calculation** with **Eq. 7.34c**:

A turbine with an **exergetic efficiency of 90%** develops **1.8×10^8 kW·h** of electricity annually. The cost rate of owning and operating the turbine is **$\$5 \times 10^5$ per year**. The unit cost based on exergy of the entering high-pressure steam is **0.02 $\$/\text{kW}\cdot\text{h}$** . Substituting values into **Eq. 7.34c**

$$c_e = \frac{c_1}{\varepsilon} + \frac{\dot{Z}_t}{\dot{W}_e}$$

$$c_e = \frac{0.02 \$/\text{kW}\cdot\text{h}}{0.90} + \frac{\$5 \times 10^5 / \text{year}}{1.8 \times 10^8 \text{ kW}\cdot\text{h} / \text{year}}$$

$$c_e = 0.0222 \$/\text{kW}\cdot\text{h} + 0.0028 \$/\text{kW}\cdot\text{h} = \mathbf{0.025 \$/\text{kW}\cdot\text{h} (2.5 \text{ cents}/\text{kW}\cdot\text{h})}$$