

# **Chapter 6**

## **Using Entropy**

# Learning Outcomes

- ▶ Demonstrate understanding of key concepts related to entropy and the second law . . . including entropy transfer, entropy production, and the increase in entropy principle.
- ▶ Evaluate entropy, evaluate entropy change between two states, and analyze isentropic processes, using appropriate property tables.

# Learning Outcomes, cont.

- ▶ **Represent heat transfer** in an internally reversible process as an area on a temperature-entropy diagram.
- ▶ **Apply entropy balances** to closed systems and control volumes.
- ▶ **Evaluate isentropic efficiencies** for turbines, nozzles, compressors, and pumps.

# Introducing Entropy Change and the Entropy Balance

- ▶ Mass and energy are familiar extensive properties of systems. **Entropy** is another important **extensive property**.
- ▶ Just as mass and energy are accounted for by mass and energy balances, **entropy is accounted for by an entropy balance**.
- ▶ Like mass and energy, **entropy can be transferred across the system boundary**.

# Introducing Entropy Change and the Entropy Balance

- The entropy change and entropy balance concepts are developed using the **Clausius inequality** expressed as:

$$\oint \left( \frac{\delta Q}{T} \right)_b = -\sigma_{\text{cycle}}$$

**(Eq. 5.13)**

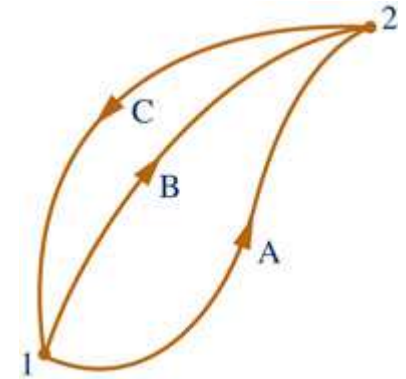
**where**

$\sigma_{\text{cycle}} = 0$  no irreversibilities present within the system  
 $\sigma_{\text{cycle}} > 0$  irreversibilities present within the system  
 $\sigma_{\text{cycle}} < 0$  impossible

**(Eq. 5.14)**

# Defining Entropy Change

- ▶ Consider **two cycles**, each composed of two internally reversible processes, **process A plus process C** and **process B plus process C**, as shown in the figure.



- ▶ Applying **Eq. 5.13** to these cycles gives,

$$\left( \int_1^2 \frac{\delta Q}{T} \right)_A + \left( \int_2^1 \frac{\delta Q}{T} \right)_C = -\sigma_{\text{cycle}}^0$$

$$\left( \int_1^2 \frac{\delta Q}{T} \right)_B + \left( \int_2^1 \frac{\delta Q}{T} \right)_C = -\sigma_{\text{cycle}}^0$$

**where**  $\sigma_{\text{cycle}}$  is zero because the cycles are composed of internally reversible processes.

# Defining Entropy Change

- ▶ Subtracting these equations:  $\left(\int_1^2 \frac{\delta Q}{T}\right)_A = \left(\int_1^2 \frac{\delta Q}{T}\right)_B$
- ▶ Since A and B are arbitrary internally reversible processes linking states 1 and 2, it follows that the **value of the integral** is independent of the particular internally reversible process and **depends on the end states only**.

# Defining Entropy Change

- ▶ Recalling (from **Sec. 1.3.3**) that a quantity is a property if, and only if, its change in value between two states is independent of the process linking the two states, we conclude that **the integral represents the change in some property of the system.**
- ▶ We call this property **entropy** and represent it by ***S***. The change in entropy is

$$S_2 - S_1 = \left( \int_1^2 \frac{\delta Q}{T} \right)_{\text{int rev}} \quad \text{(Eq. 6.2a)}$$

**where** the subscript “int rev” signals that the integral is carried out for any internally reversible process linking states 1 and 2.



# Defining Entropy Change

► **Equation 6.2a** allows the change in entropy between two states to be determined by thinking of an internally reversible process between the two states. But **since entropy is a property**, that **value of entropy change applies to *any* process between the states** – internally reversible or not.

► Entropy change is introduced by the integral of **Eq. 6.2a** for which no accompanying physical picture is given. Still, the aim of Chapter 6 is to demonstrate that **entropy not only has physical significance but also is essential for thermodynamic analysis.**

## Entropy Facts

- ▶ Entropy is an extensive property.
- ▶ Like any other extensive property, the change in entropy can be positive, negative, or zero:

$$S_2 - S_1: \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

- ▶ By inspection of **Eq. 6.2a**, units for entropy  $S$  are kJ/K.
- ▶ Units for *specific* entropy  $s$  are kJ/kg·K.

# Entropy Facts

- ▶ For problem solving, specific entropy values are provided in **Tables A-2** through **A-18**. Values for specific entropy are obtained from these tables using the same procedures as for specific volume, internal energy, and enthalpy, including use of

$$\begin{aligned} s &= (1 - x)s_f + xs_g \\ &= s_f + x(s_g - s_f) \end{aligned}$$

**(Eq. 6.4)**

for **two-phase liquid-vapor mixtures**, and

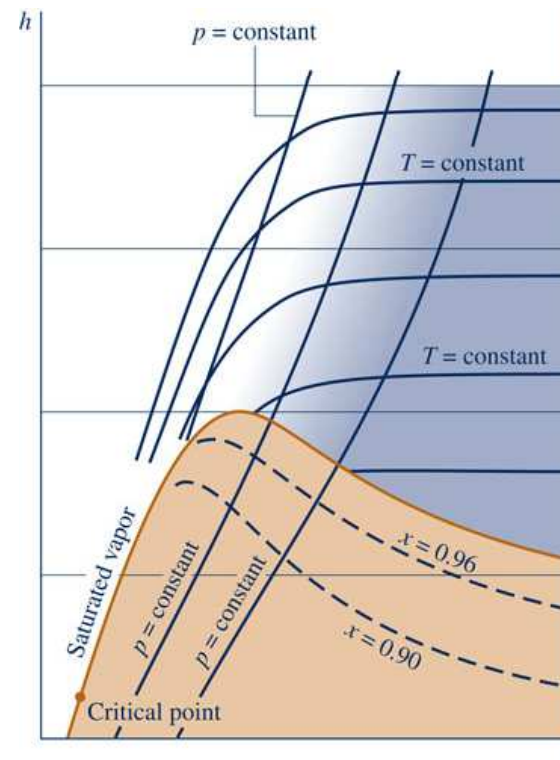
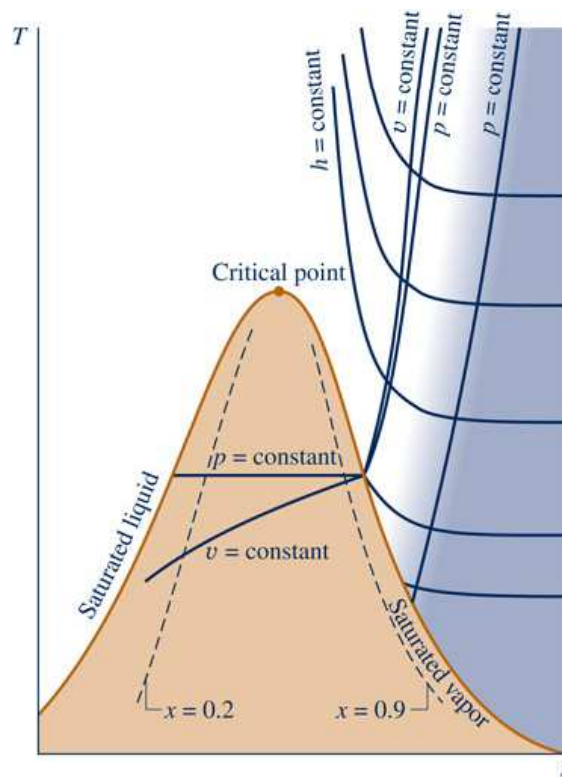
$$s(T, p) \approx s_f(T)$$

**(Eq. 6.5)**

for **liquid water**, each of which is similar in form to expressions introduced in Chap. 3 for evaluating  $v$ ,  $u$ , and  $h$ .

# Entropy Facts

- ▶ For problem solving, states often are shown on property diagrams having specific entropy as a coordinate: the **temperature-entropy** and **enthalpy-entropy** (Mollier) **diagrams** shown here



# Entropy and Heat Transfer

- ▶ By inspection of **Eq. 6.2a**, the defining equation for entropy change on a differential basis is

$$dS = \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \quad \text{(Eq. 6.2b)}$$

- ▶ **Equation 6.2b** indicates that when a closed system undergoing an internally reversible process receives energy by heat transfer, the system experiences an increase in entropy. Conversely, when energy is removed by heat transfer, the entropy of the system decreases. From these considerations, we say that **entropy transfer accompanies heat transfer**. The **direction of the entropy transfer is the same as the heat transfer**.

# Entropy and Heat Transfer

- ▶ In an internally reversible, adiabatic process (no heat transfer), entropy remains constant. Such a **constant-entropy process** is called an **isentropic** process.
- ▶ On rearrangement, **Eq. 6.2b** gives

$$(\delta Q)_{\text{int rev}} = T dS$$

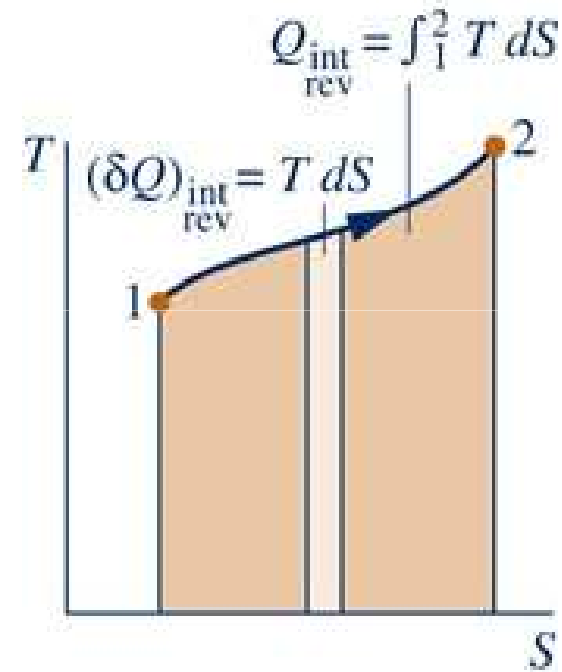
Integrating from state 1 to state 2,

$$Q_{\text{int rev}} = \int_1^2 T dS$$

**(Eq. 6.23)**

# Entropy and Heat Transfer

From this it follows that an **energy transfer by heat to a closed system** during an internally reversible process is **represented by an area on a temperature-entropy diagram**:



# Entropy Balance for Closed Systems

- ▶ The entropy balance for closed systems can be developed using the Clausius inequality expressed as **Eq. 5.13** and the defining equation for entropy change, **Eq. 6.2a**. The result is

$$\underbrace{S_2 - S_1}_{\text{entropy change}} = \underbrace{\int_1^2 \left( \frac{\delta Q}{T} \right)_b}_{\text{entropy transfer}} + \underbrace{\sigma}_{\text{entropy production}} \quad (\text{Eq. 6.24})$$

**where** the subscript **b** indicates the integral is evaluated at the system boundary.

- ▶ In accord with the interpretation of  $\sigma_{\text{cycle}}$  in the Clausius inequality, **Eq. 5.14**, the **value of  $\sigma$**  in **Eq. 6.24** adheres to the following **interpretation**

$$\sigma: \begin{cases} = 0 & \text{(no irreversibilities present within the system)} \\ > 0 & \text{(irreversibilities present within the system)} \\ < 0 & \text{(impossible)} \end{cases}$$



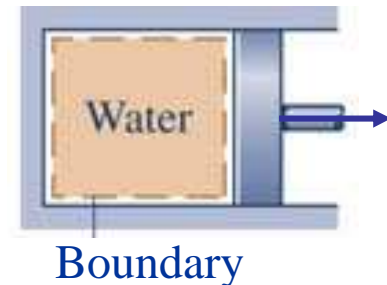
# Entropy Balance for Closed Systems

- ▶ That  $\sigma$  has a value of zero when there are no internal irreversibilities and is positive when irreversibilities are present within the system leads to the interpretation that  **$\sigma$  accounts for entropy produced** (or generated) within the system by action of irreversibilities.
- ▶ Expressed in words, the entropy balance is

$$\left[ \begin{array}{c} \text{change in the amount} \\ \text{of entropy contained} \\ \text{within the system} \\ \text{during some} \\ \text{time interval} \end{array} \right] = \left[ \begin{array}{c} \text{net amount of} \\ \text{entropy transferred in} \\ \text{across the system boundary} \\ \text{accompanying heat transfer} \\ \text{during some time interval} \end{array} \right] + \left[ \begin{array}{c} \text{amount of} \\ \text{entropy produced} \\ \text{within the system} \\ \text{during some} \\ \text{time interval} \end{array} \right]$$

# Entropy Balance for Closed Systems

**Example:** One kg of water vapor contained within a piston-cylinder assembly, **initially at 5 bar, 400°C**, undergoes an **adiabatic expansion** to a state where pressure is **1 bar** and the temperature is **(a) 200°C**, **(b) 100°C**. Using the entropy balance, determine the nature of the process in each case.



- Since the expansion occurs adiabatically, **Eq. 6.24** reduces to give

$$S_2 - S_1 = \int \left( \frac{\delta Q}{T} \right)_b + \sigma \rightarrow m(s_2 - s_1) = \sigma \quad (1)$$

where  $m = 1 \text{ kg}$  and **Table A-4** gives  $s_1 = 7.7938 \text{ kJ/kg}\cdot\text{K}$ .

## Entropy Balance for Closed Systems

**(a) Table A-4** gives,  $s_2 = 7.8343 \text{ kJ/kg}\cdot\text{K}$ . Thus **Eq. (1)** gives

$$\sigma = (1 \text{ kg})(7.8343 - 7.7938) \text{ kJ/kg}\cdot\text{K} = \mathbf{0.0405 \text{ kJ/K}}$$

Since  $\sigma$  is positive, irreversibilities are present within the system during expansion (a).

**(b) Table A-4** gives,  $s_2 = 7.3614 \text{ kJ/kg}\cdot\text{K}$ . Thus **Eq. (1)** gives

$$\sigma = (1 \text{ kg})(7.3614 - 7.7938) \text{ kJ/kg}\cdot\text{K} = \mathbf{-0.4324 \text{ kJ/K}}$$

Since  $\sigma$  is negative, expansion (b) is impossible: it cannot occur *adiabatically*.

# Entropy Balance for Closed Systems

More about expansion (b): Considering Eq. 6.24

$$\begin{array}{ccc} S_2 - S_1 = \int_1^2 \left( \frac{\delta Q}{T} \right)_b + \sigma \\ \hline \text{entropy} & \text{entropy} & \text{entropy} \\ \text{change} & \text{transfer} & \text{production} \end{array}$$
$$< 0 = < 0 + \geq 0$$

- ▶ Since  $\sigma$  cannot be negative and
- ▶ For expansion (b)  $\Delta S$  is negative, then
- ▶ By inspection the integral must be negative and so heat transfer *from* the system *must* occur in expansion (b).

## Entropy Rate Balance for Closed Systems

- On a time rate basis, the **closed system entropy rate balance** is

$$\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma} \quad \text{(Eq. 6.28)}$$

where

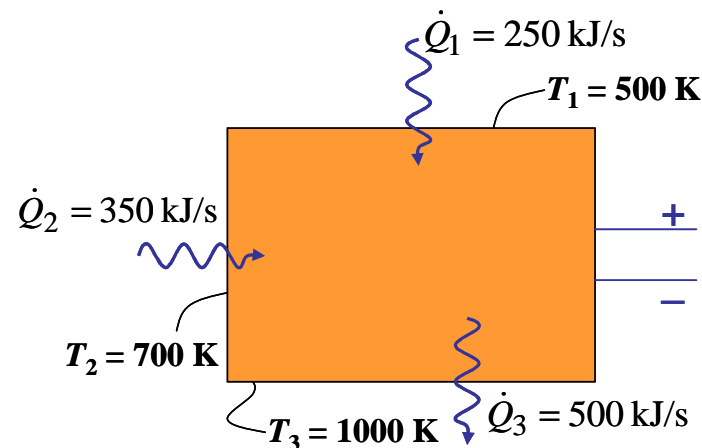
$\frac{dS}{dt}$  = the time rate of change of the entropy of the system

$\frac{\dot{Q}_j}{T_j}$  = the time rate of entropy transfer through the portion of the boundary whose temperature is  $T_j$

$\dot{\sigma}$  = time rate of entropy production due to irreversibilities within the system

## Entropy Rate Balance for Closed Systems

**Example:** An inventor claims that the device shown generates electricity at a rate of 100 kJ/s while *receiving* a heat transfer of energy at a rate of **250 kJ/s** at a temperature of **500 K**, *receiving* a second heat transfer at a rate of **350 kJ/s** at **700 K**, and *discharging* energy by heat transfer at a rate of **500 kJ/s** at a temperature of **1000 K**. Each heat transfer is positive in the direction of the accompanying arrow. For operation at steady state, evaluate this claim.



## Entropy Rate Balance for Closed Systems

- ▶ Applying an **energy rate balance at steady state**

$$\frac{dE}{dt} = 0 = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3 - \dot{W}_e$$

Solving  $\dot{W}_e = 250 \text{ kJ/s} + 350 \text{ kJ/s} - 500 \text{ kJ/s} = \mathbf{100 \text{ kJ/s}}$

The claim is in accord with the first law of thermodynamics.

- ▶ Applying an **entropy rate balance at steady state**

$$\frac{dS}{dt} = 0 = \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3} + \dot{\sigma}$$

Solving

$$\dot{\sigma} = -\left( \frac{250 \text{ kJ/s}}{500 \text{ K}} + \frac{350 \text{ kJ/s}}{700 \text{ K}} - \frac{500 \text{ kJ/s}}{1000 \text{ K}} \right)$$
$$\dot{\sigma} = -(0.5 + 0.5 - 0.5) \frac{\text{kJ/s}}{\text{K}} = \mathbf{-0.5 \frac{\text{kJ/s}}{\text{K}}}$$

Since  $\dot{\sigma}$  is negative, the claim is **not** in accord with the second law of thermodynamics and is therefore denied.

## Entropy Rate Balance for Control Volumes

- ▶ Like mass and energy, **entropy can be transferred into or out of a control volume by streams of matter.**
- ▶ Since this is the principal difference between the closed system and control volume entropy rate balances, the **control volume form** can be obtained by modifying the closed system form to **account for such entropy transfer.** The result is

$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

rate of entropy change		rates of entropy transfer		rate of entropy production
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**(Eq. 6.34)**

**where**  $\dot{m}_i s_i$  and  $\dot{m}_e s_e$  account, respectively, for **rates of entropy transfer accompanying mass flow** at inlets  $i$  and exits  $e$ .



## Entropy Rate Balance for Control Volumes

- ▶ For control volumes at **steady state**, **Eq. 6.34** reduces to give

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv} \quad \text{(Eq. 6.36)}$$

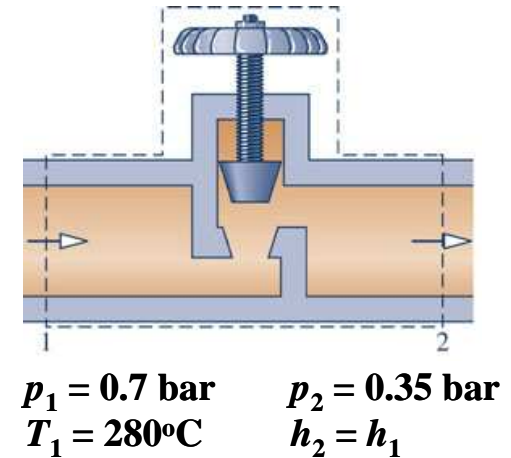
- ▶ For a **one-inlet, one-exit control volume at steady state**, **Eq. 6.36** reduces to give

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \quad \text{(Eq. 6.37)}$$

**where** 1 and 2 denote the inlet and exit, respectively, and  $\dot{m}$  is the common mass flow rate at these locations.

# Entropy Rate Balance for Control Volumes

**Example:** Water vapor enters a valve at **0.7 bar**, **280°C** and exits at **0.35 bar**. **(a)** If the water vapor undergoes a throttling process, determine the rate of entropy production within the valve, in kJ/K per kg of water vapor flowing. **(b)** What is the source of entropy production in this case?



**(a)** For a throttling process, there is no significant heat transfer. Thus, **Eq. 6.37** reduces to

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \rightarrow 0 = \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

## Entropy Rate Balance for Control Volumes

**Solving**

$$\frac{\dot{\sigma}_{\text{cv}}}{\dot{m}} = s_2 - s_1$$

From **Table A-4**,  $h_1 = 3035.0 \text{ kJ/kg}$ ,  $s_1 = 8.3162 \text{ kJ/kg}\cdot\text{K}$ .

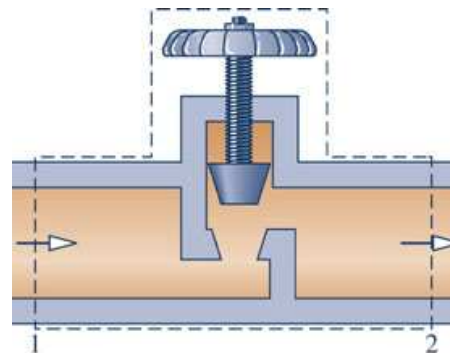
For a throttling process,  $h_2 = h_1$  (**Eq. 4.22**). Interpolating in **Table A-4** at 0.35 bar and  $h_2 = 3035.0 \text{ kJ/kg}$ ,  
 $s_2 = 8.6295 \text{ kJ/kg}\cdot\text{K}$ .

**Finally**  $\frac{\dot{\sigma}_{\text{cv}}}{\dot{m}} = (8.6295 - 8.3162) \text{ kJ/kg}\cdot\text{K} = 0.3133 \text{ kJ/kg}\cdot\text{K}$

**(b)** Selecting from the list of irreversibilities provided in **Sec. 5.3.1**, the source of the entropy production here is the *unrestrained expansion to a lower pressure* undergone by the water vapor.

## Entropy Rate Balance for Control Volumes

**Comment:** The value of the entropy production for a single component such as the throttling valve considered here often does not have much significance by itself. The **significance of the entropy production of any component is normally determined through comparison with the entropy production values of other components** combined with that component to form an integrated system. **Reducing irreversibilities of components with the highest entropy production rates may lead to improved thermodynamic performance of the integrated system.**



## Calculating Entropy Change

- ▶ The property data provided in **Tables A-2** through **A-18**, similar compilations for other substances, and numerous important relations among such properties are established using the ***Tds* equations**. When expressed on a **unit mass basis**, these equations are

$$T ds = du + p dv \quad \text{(Eq. 6.10a)}$$

$$T ds = dh - v dp \quad \text{(Eq. 6.10b)}$$

# Calculating Entropy Change

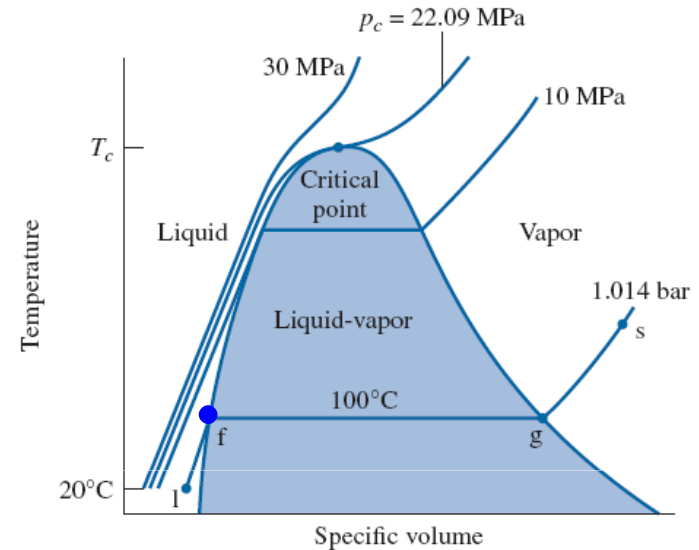
- ▶ As an application, consider a change in phase from saturated liquid to saturated vapor at constant pressure.
- ▶ Since pressure is constant, **Eq. 6.10b** reduces to give

$$ds = \frac{dh}{T}$$

- ▶ Then, because temperature is also constant during the phase change

$$s_g - s_f = \frac{h_g - h_f}{T} \quad \text{(Eq. 6.12)}$$

This relationship is applied in property tables for tabulating  $(s_g - s_f)$  from known values of  $(h_g - h_f)$ .



# Calculating Entropy Change

► **For example**, consider water vapor at 100°C (373.15 K). From **Table A-2**,  $(h_g - h_f) = 2257.1 \text{ kJ/kg}$ .

**Thus**

$$(s_g - s_f) = (2257.1 \text{ kJ/kg})/373.15 \text{ K} = 6.049 \text{ kJ/kg}\cdot\text{K}$$

which agrees with the value from **Table A-2**, as expected.

► Next, the  $TdS$  equations are applied to two additional cases: **substances modeled as incompressible** and **gases modeled as ideal gases**.

## Calculating Entropy Change of an Incompressible Substance

- ▶ The **incompressible substance model** assumes the **specific volume is constant** and **specific internal energy depends solely on temperature**:  $u = u(T)$ . Thus,  $du = c(T)dT$ , where  $c$  denotes specific heat.
- ▶ With these relations, **Eq. 6.10a** reduces to give

$$ds = \frac{c(T)dT}{T} + \frac{pdv^0}{T} = \frac{c(T)dT}{T}$$

- ▶ On integration, the **change in specific entropy** is

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c(T)}{T} dT$$

- ▶ When the **specific heat is constant**

$$s_2 - s_1 = c \ln \frac{T_2}{T_1} \quad (\text{incompressible, constant } c)$$

**(Eq. 6.13)**



## Calculating Entropy Change of an Ideal Gas

- ▶ The **ideal gas model** assumes pressure, specific volume and temperature are related by  $p v = RT$ . Also, **specific internal energy and specific enthalpy each depend solely on temperature**:  $u = u(T)$ ,  $h = h(T)$ , giving  $du = c_v dT$  and  $dh = c_p dT$ , respectively.
- ▶ Using these relations and integrating, the  **$Tds$  equations** give, respectively

$$ds = \frac{du}{T} + \frac{p}{T} dv$$

$$ds = c_v(T) \frac{dT}{T} + R \frac{dv}{v}$$

$$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

**(Eq. 6.17)**

$$ds = \frac{dh}{T} - \frac{v}{T} dp$$

$$ds = c_p(T) \frac{dT}{T} - R \frac{dp}{p}$$

$$s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} c_p(T) \frac{dT}{T} - R \ln \frac{p_2}{p_1}$$

**(Eq. 6.18)**

## Calculating Entropy Change of an Ideal Gas

- ▶ Since these particular equations give entropy change on a unit of mass basis, the constant  $R$  is determined from

$$R = \bar{R} / M.$$

- ▶ Since  $c_v$  and  $c_p$  are functions of temperature for ideal gases, such functional relations are required to perform the integration of the first term on the right of **Eqs. 6.17** and **6.18**.
- ▶ For several gases modeled as ideal gases, including air, CO<sub>2</sub>, CO, O<sub>2</sub>, N<sub>2</sub>, and water vapor, the **evaluation of entropy change can be reduced to a convenient tabular approach using the variable  $s^\circ$**  defined by

$$s^\circ(T) = \int_{T'}^T \frac{c_p(T)}{T} dT \quad \text{(Eq. 6.19)}$$

**where**  $T'$  is an arbitrary reference temperature.

## Calculating Entropy Change of an Ideal Gas

- Using  $s^\circ$ , the integral term of **Eq. 6.18** can be expressed as

$$\begin{aligned}\int_{T_1}^{T_2} c_p \frac{dT}{T} &= \int_{T'}^{T_2} c_p \frac{dT}{T} - \int_{T'}^{T_1} c_p \frac{dT}{T} \\ &= s^\circ(T_2) - s^\circ(T_1)\end{aligned}$$

- Accordingly, **Eq. 6.18** becomes

$$s(T_2, p_2) - s(T_1, p_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1} \quad \text{(Eq. 6.20a)}$$

or on a **per mole basis** as

$$\bar{s}(T_2, p_2) - \bar{s}(T_1, p_1) = \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln \frac{p_2}{p_1} \quad \text{(Eq. 6.20b)}$$

- For air, **Table A-22** provides  $s^\circ$  in units of kJ/kg·K. For the other gases mentioned, **Table A-23** provides  $\bar{s}^\circ$  in units of kJ/kmol·K.

# Calculating Entropy Change of an Ideal Gas

**Example:** Determine the change in specific entropy, in kJ/kg·K, of air as an ideal gas undergoing a process from  $T_1 = 300 \text{ K}$ ,  $p_1 = 1 \text{ bar}$  to  $T_2 = 1420 \text{ K}$ ,  $p_2 = 5 \text{ bar}$ .

► From **Table A-22**, we get  $s_1^\circ = 1.70203$  and  $s_2^\circ = 3.37901$ , each in kJ/kg·K. Substituting into **Eq. 6.20a**

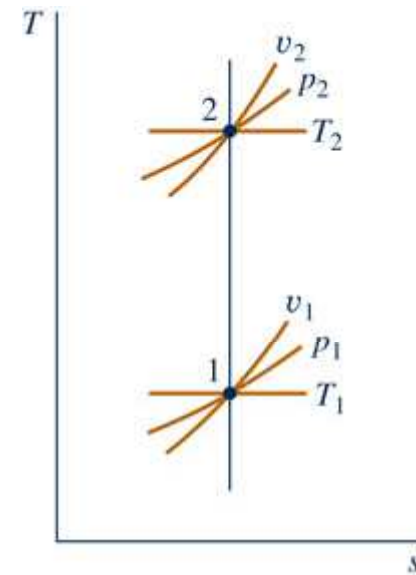
$$s_2 - s_1 = (3.37901 - 1.70203) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - \left( \frac{8.314}{28.97} \right) \left( \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \ln \left( \frac{5 \text{ bar}}{1 \text{ bar}} \right) = 1.215 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Ideal Gas Properties of Air

Table A-22				$T(\text{K}), h \text{ and } u(\text{kJ/kg}), s^\circ (\text{kJ/kg}\cdot\text{K})$							
				when $\Delta s = 0$				when $\Delta s = 0$			
$T$	$h$	$u$	$s^\circ$	$p_r$	$v_r$	$T$	$h$	$u$	$s^\circ$	$p_r$	$v_r$
250	250.05	178.28	1.51917	0.7329	979.	1400	1515.42	1113.52	3.36200	450.5	8.919
260	260.09	185.45	1.55848	0.8405	887.8	1420	1539.44	1131.77	3.37901	478.0	8.526
270	270.11	192.60	1.59634	0.9590	808.0	1440	1563.51	1150.13	3.39586	506.9	8.153
280	280.13	199.75	1.63279	1.0889	738.0	1460	1587.63	1168.49	3.41247	537.1	7.801
285	285.14	203.33	1.65055	1.1584	706.1	1480	1611.79	1186.95	3.42892	568.8	7.468
290	290.16	206.91	1.66802	1.2311	676.1	1500	1635.97	1205.41	3.44516	601.9	7.152
295	295.17	210.49	1.68515	1.3068	647.9	1520	1660.23	1223.87	3.46120	636.5	6.854
300	300.19	214.07	1.70203	1.3860	621.2	1540	1684.51	1242.43	3.47712	672.8	6.569
305	305.22	217.67	1.71865	1.4686	596.0	1560	1708.82	1260.99	3.49276	710.5	6.301
310	310.24	221.25	1.73498	1.5546	572.3	1580	1733.17	1279.65	3.50829	750.0	6.046

## Calculating Entropy Change of an Ideal Gas

► **Table A-22** provides additional data for air modeled as an ideal gas. These values, denoted by  $p_r$  and  $v_r$ , refer only to **two states having the same specific entropy**. This case has important applications, and is shown in the figure.



# Calculating Entropy Change of an Ideal Gas

- When  $s_2 = s_1$ , the following equation relates  $T_1$ ,  $T_2$ ,  $p_1$ , and  $p_2$

$$\frac{p_2}{p_1} = \frac{p_r(T_2)}{p_r(T_1)} \quad (s_1 = s_2, \text{ air only}) \quad \text{(Eq. 6.41)}$$

where  $p_r(T)$  is read from **Table A-22**, as appropriate.

Ideal Gas Properties of Air

Table A-22				$T(\text{K}), h$ and $u(\text{kJ/kg}), s^\circ (\text{kJ/kg}\cdot\text{K})$							
				when $\Delta s = 0$				when $\Delta s = 0$			
$T$	$h$	$u$	$s^\circ$	$p_r$	$v_r$	$T$	$h$	$u$	$s^\circ$	$p_r$	$v_r$
250	250.05	178.28	1.51917	0.7329	979.	1400	1515.42	1113.52	3.36200	450.5	8.919
260	260.09	185.45	1.55848	0.8405	887.8	1420	1539.44	1131.77	3.37901	478.0	8.526
270	270.11	192.60	1.59634	0.9590	808.0	1440	1563.51	1150.13	3.39586	506.9	8.153
280	280.13	199.75	1.63279	1.0889	738.0	1460	1587.63	1168.49	3.41247	537.1	7.801
285	285.14	203.33	1.65055	1.1584	706.1	1480	1611.79	1186.95	3.42892	568.8	7.468
290	290.16	206.91	1.66802	1.2311	676.1	1500	1635.97	1205.41	3.44516	601.9	7.152
295	295.17	210.49	1.68515	1.3068	647.9	1520	1660.23	1223.87	3.46120	636.5	6.854
300	300.19	214.07	1.70203	1.3860	621.2	1540	1684.51	1242.43	3.47712	672.8	6.569
305	305.22	217.67	1.71865	1.4686	596.0	1560	1708.82	1260.99	3.49276	710.5	6.301
310	310.24	221.25	1.73498	1.5546	572.3	1580	1733.17	1279.65	3.50829	750.0	6.046

# Calculating Entropy Change of an Ideal Gas

- When  $s_2 = s_1$ , the following equation relates  $T_1$ ,  $T_2$ ,  $v_1$ , and  $v_2$

$$\frac{v_2}{v_1} = \frac{v_r(T_2)}{v_r(T_1)} \quad (s_1 = s_2, \text{ air only}) \quad \text{(Eq. 6.42)}$$

where  $v_r(T)$  is read from **Table A-22**, as appropriate.

Ideal Gas Properties of Air

Table A-22				$T(\text{K}), h$ and $u(\text{kJ/kg}), s^\circ (\text{kJ/kg}\cdot\text{K})$							
				when $\Delta s = 0$				when $\Delta s = 0$			
$T$	$h$	$u$	$s^\circ$	$p_r$	$v_r$	$T$	$h$	$u$	$s^\circ$	$p_r$	$v_r$
250	250.05	178.28	1.51917	0.7329	979.	1400	1515.42	1113.52	3.36200	450.5	8.919
260	260.09	185.45	1.55848	0.8405	887.8	1420	1539.44	1131.77	3.37901	478.0	8.526
270	270.11	192.60	1.59634	0.9590	808.0	1440	1563.51	1150.13	3.39586	506.9	8.153
280	280.13	199.75	1.63279	1.0889	738.0	1460	1587.63	1168.49	3.41247	537.1	7.801
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305	305.22	217.67	1.71865	1.4686	596.0	1560	1708.82	1260.99	3.49276	710.5	6.301
310	310.24	221.25	1.73498	1.5546	572.3	1580	1733.17	1279.65	3.50829	750.0	6.046

# Entropy Change of an Ideal Gas Assuming Constant Specific Heats

- ▶ When the specific heats  $c_v$  and  $c_p$  are assumed constant, Eqs. 6.17 and 6.18 reduce, respectively, to

(Eq. 6.17)

$$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$



$$s(T_2, v_2) - s(T_1, v_1) = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

(Eq. 6.21)

(Eq. 6.18)

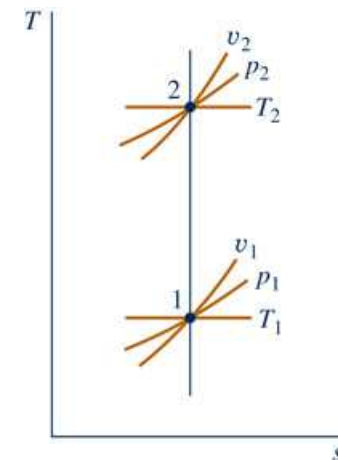
$$s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} c_p(T) \frac{dT}{T} - R \ln \frac{p_2}{p_1}$$



$$s(T_2, p_2) - s(T_1, p_1) = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

(Eq. 6.22)

- ▶ These expressions have many applications. In particular, they can be applied to develop relations among  $T$ ,  $p$ , and  $v$  at two states having the same specific entropy as shown in the figure.





# Entropy Change of an Ideal Gas Assuming Constant Specific Heats

- ▶ Since  $s_2 = s_1$ , **Eqs. 6.21** and **6.22** become

$$0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$0 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

- ▶ With the ideal gas relations **where  $k$  is the specific ratio**

$$c_p(T) = \frac{kR}{k-1}$$

$$c_v(T) = \frac{R}{k-1}$$

- ▶ These equations can be solved, respectively, to give

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{(k-1)/k} \quad (s_1 = s_2, \text{ constant } k)$$

**(Eq. 6.43)**

$$\frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{k-1} \quad (s_1 = s_2, \text{ constant } k)$$

**(Eq. 6.44)**

- ▶ Eliminating the temperature ratio gives

$$\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^k \quad (s_1 = s_2, \text{ constant } k)$$

**(Eq. 6.45)**

# Calculating Entropy Change of an Ideal Gas

**Example:** Air undergoes a process from  $T_1 = 620 \text{ K}$ ,  $p_1 = 12 \text{ bar}$  to a final state where  $s_2 = s_1$ ,  $p_2 = 1.4 \text{ bar}$ . Employing the ideal gas model, determine the final temperature  $T_2$ , in K. Solve using (a)  $p_r$  data from Table A-22 and (b) a constant specific heat ratio  $k$  evaluated at 620 K from Table A-20:  $k = 1.374$ . Comment.

(a) With Eq. 6.41 and  $p_r(T_1) = 18.36$  from Table A-22

$$p_r(T_2) = p_r(T_1) \left( \frac{p_2}{p_1} \right) = 18.36 \left( \frac{1.4 \text{ bar}}{12 \text{ bar}} \right) = 2.142$$

Interpolating in Table A-22,  $T_2 = 339.7 \text{ K}$

Ideal Gas Properties of Air

Table A-22		$T(\text{K}), h \text{ and } u(\text{kJ/kg}), s^\circ (\text{kJ/kg}\cdot\text{K})$									
when $\Delta s = 0$						when $\Delta s = 0$					
$T$	$h$	$u$	$s^\circ$	$p_r$	$v_r$	$T$	$h$	$u$	$s^\circ$	$p_r$	$v_r$
315	315.27	224.85	1.75106	1.6442	549.8	600	607.02	434.78	2.40902	16.28	105.8
320	320.29	228.42	1.76690	1.7375	528.6	610	617.53	442.42	2.42644	17.30	101.2
325	325.31	232.02	1.78249	1.8345	508.4	620	628.07	450.09	2.44356	18.36	96.92
330	330.34	235.61	1.79783	1.9352	489.4	630	638.63	457.78	2.46048	19.84	92.84
340	340.42	242.82	1.82790	2.149	454.1	640	649.22	465.50	2.47716	20.64	88.99
350	350.49	250.02	1.85708	2.379	422.2	650	659.84	473.25	2.49364	21.86	85.34

## Calculating Entropy Change of an Ideal Gas

(b) With **Eq. 6.43**

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{(k-1)/k} = 620 \text{ K} \left( \frac{1.4 \text{ bar}}{12 \text{ bar}} \right)^{(0.374/1.374)}$$

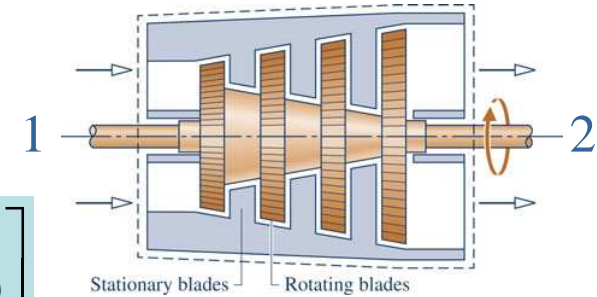
$$T_2 = 345.5 \text{ K}$$

**Comment:** The approach of (a) accounts for variation of specific heat with temperature but the approach of (b) does not. With a  $k$  value more representative of the temperature interval, the value obtained in (b) using **Eq. 6.43** would be in better agreement with that obtained in (a) with **Eq. 6.41**.

# Isentropic Turbine Efficiency

- ▶ For a turbine, the energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$



- ▶ If the change in kinetic energy of flowing matter is negligible,  $\frac{1}{2}(V_1^2 - V_2^2)$  drops out.
- ▶ If the change in potential energy of flowing matter is negligible,  $g(z_1 - z_2)$  drops out.
- ▶ If the heat transfer with surroundings is negligible,  $\dot{Q}_{cv}$  drops out.

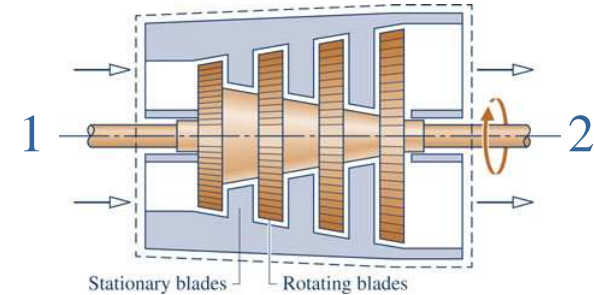
$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2$$

where

the left side is **work developed** per unit of mass flowing.

# Iisentropic Turbine Efficiency

- ▶ For a turbine, the entropy rate balance reduces to



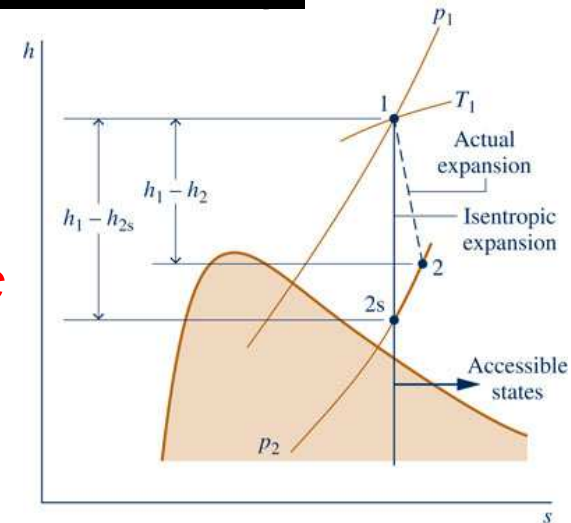
$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

- ▶ If the heat transfer with surroundings is negligible,  $\dot{Q}_j$  drops out.

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \geq 0$$

# Isentropic Turbine Efficiency

- ▶ Since the rate of entropy production cannot be negative, **the only turbine exit states that can be attained in an *adiabatic* expansion** are those with  $s_2 \geq s_1$ . This is shown on the Mollier diagram to the right.

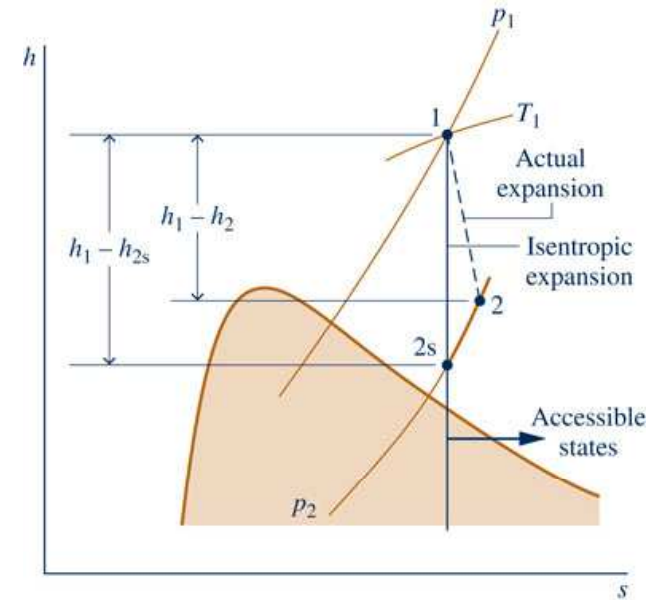


- ▶ The state labeled 2s on the figure would be attained only in an isentropic expansion from the inlet state to the specified exit pressure – that is, **2s would be attained only in the absence of internal irreversibilities**. By inspection of the figure, **the maximum theoretical value for the turbine work per unit of mass flowing** is developed in such an internally reversible, adiabatic expansion:

$$\left( \frac{\dot{W}_{cv}}{\dot{m}} \right)_s = h_1 - h_{2s}$$

# Isentropic Turbine Efficiency

► The **isentropic turbine efficiency** is the ratio of the actual turbine work to the maximum theoretical work, each per unit of mass flowing:

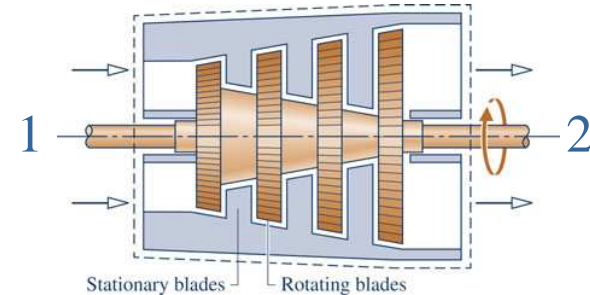


$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{(\dot{W}_{cv}/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

**(Eq. 6.46)**

# Isentropic Turbine Efficiency

**Example:** Water vapor enters a turbine at  $p_1 = 5 \text{ bar}$ ,  $T_1 = 320^\circ\text{C}$  and exits at  $p_2 = 1 \text{ bar}$ . The work developed is measured as **271 kJ per kg** of water vapor flowing. Applying **Eq. 6.46**, determine the isentropic turbine efficiency.



- ▶ From **Table A-4**,  $h_1 = 3105.6 \text{ kJ/kg}$ ,  $s_1 = 7.5308 \text{ kJ/kg}$ .  
With  $s_{2s} = s_1$ , interpolation in **Table A-4** at a pressure of 1 bar gives  $h_{2s} = 2743.0 \text{ kJ/kg}$ . Substituting values into **Eq. 6.46**

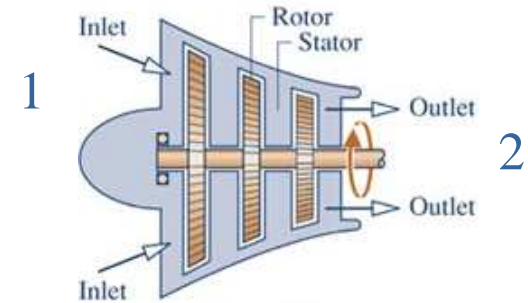
$$\eta_t = \frac{\dot{W}_{cv} / \dot{m}}{h_1 - h_{2s}} = \frac{271 \text{ kJ/kg}}{(3105.6 - 2743.0) \text{ kJ/kg}} = 0.75 \text{ (75\%)}$$



# Isentropic Compressor and Pump Efficiencies

- ▶ For a compressor the energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right]$$



- ▶ If the change in kinetic energy of flowing matter is negligible,  $\frac{1}{2}(V_1^2 - V_2^2)$  drops out.
- ▶ If the change in potential energy of flowing matter is negligible,  $g(z_1 - z_2)$  drops out.
- ▶ If the heat transfer with surroundings is negligible,  $\dot{Q}_{cv}$  drops out.

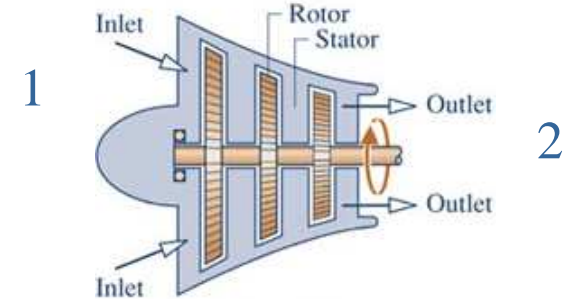
$$\left( -\frac{\dot{W}_{cv}}{\dot{m}} \right) = h_2 - h_1$$

where

the left side is **work input** per unit of mass flowing.

# Isentropic Compressor and Pump Efficiencies

- ▶ For a compressor the entropy rate balance reduces to



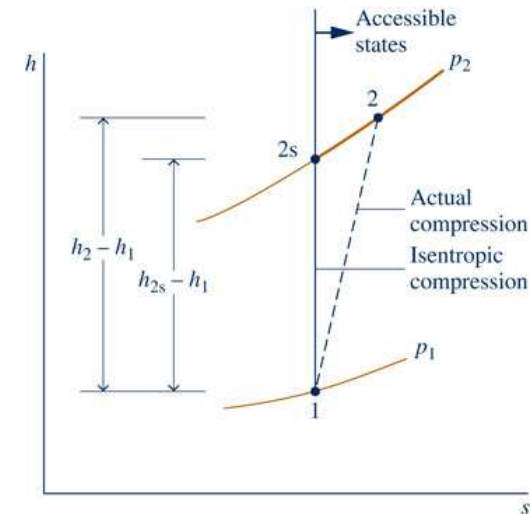
$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

- ▶ If the heat transfer with surroundings is negligible,  $\dot{Q}_j$  drops out.

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \geq 0$$

# Isentropic Compressor and Pump Efficiencies

- ▶ Since the rate of entropy production cannot be negative, **the only compressor exit states that can be attained in an *adiabatic* compression** are those with  $s_2 \geq s_1$ . This is shown on the Mollier diagram to the right.

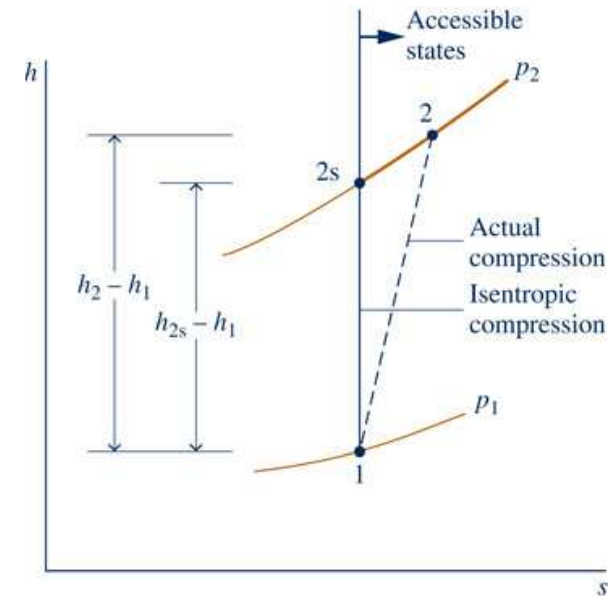


- ▶ The state labeled **2s** on the figure would be attained only in an isentropic compression from the inlet state to the specified exit pressure – that is, state **2s would be attained only in the absence of internal irreversibilities**. By inspection of the figure, the **minimum theoretical value for the compressor work input per unit of mass flowing** is for such an internally reversible, adiabatic compression:

$$\left( -\frac{\dot{W}_{cv}}{\dot{m}} \right)_s = h_{2s} - h_1$$

# Isentropic Compressor and Pump Efficiencies

- ▶ The **isentropic compressor efficiency** is the ratio of the minimum theoretical work input to the actual work input, each per unit of mass flowing:



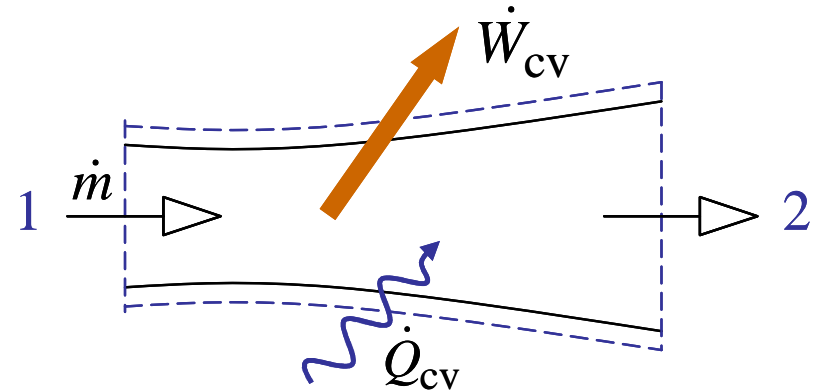
$$\eta_c = \frac{(-\dot{W}_{cv}/\dot{m})_s}{(-\dot{W}_{cv}/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

**(Eq. 6.48)**

- ▶ An **isentropic pump efficiency** is defined similarly.

# Heat Transfer and Work in Internally Reversible Steady-State Flow Processes

- ▶ Consider a **one-inlet, one-exit control volume at steady state**:
- ▶ Compressors, pumps, and other devices commonly encountered in engineering practice are included in this class of control volumes.
- ▶ The **objective** is to introduce expressions for the heat transfer rate  $\dot{Q}_{cv} / \dot{m}$  and work rate  $\dot{W}_{cv} / \dot{m}$  in the absence of internal irreversibilities. The resulting expressions have important applications.



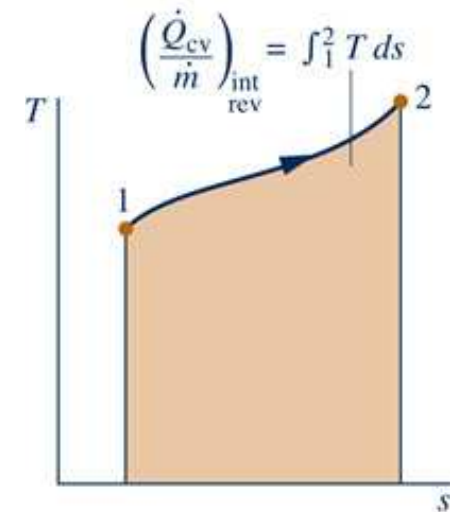
# Heat Transfer and Work in Internally Reversible Steady-State Flow Processes

- ▶ In agreement with the discussion of energy transfer by heat to a closed system during an internally reversible process (**Sec. 6.6.1**), in the present application we have

$$\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\text{int rev}} = \int_1^2 T ds \quad \text{(Eq. 6.49)}$$

**where** the subscript “**int rev**” signals that the expression applies only in the absence of internal irreversibilities.

- ▶ As shown by the figure, when the states visited by a unit mass passing from inlet to exit without internal irreversibilities are described by a curve on a ***T-s* diagram**, the **heat transfer** per unit of mass flowing is represented by the area under the curve.



# Heat Transfer and Work in Internally Reversible Steady-State Flow Processes

- ▶ Neglecting kinetic and potential energy effects, an energy rate balance for the control volume reduces to

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{int, rev} = \left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{int, rev} + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)$$

- ▶ With **Eq. 6.49**, this becomes  $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{int, rev} = \int_1^2 T ds + (h_1 - h_2)$  **(1)**

- ▶ Since **internal irreversibilities are assumed absent**, each unit of mass visits a sequence of equilibrium states as it passes from inlet to exit. Entropy, enthalpy, and pressure changes are therefore related by the ***Tds*** equation, **Eq. 6.10b**:

$$T ds = dh - v dp$$

# Heat Transfer and Work in Internally Reversible Steady-State Flow Processes

- ▶ Integrating from inlet to exit:  $\int_1^2 T ds = (h_2 - h_1) - \int_1^2 v dp$
- ▶ With this relation **Eq. (1)** becomes

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\text{int rev}} = -\int_1^2 v dp \quad \text{(Eq. 6.51b)}$$

- ▶ If the **specific volume remains approximately constant**, as in many applications with liquids, **Eq. 6.51b** becomes

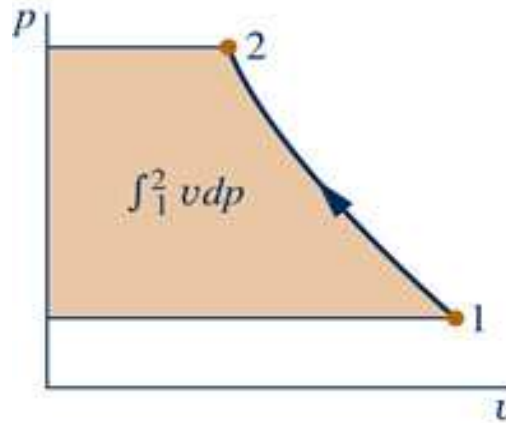
$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\text{int rev}} = -v(p_2 - p_1) \quad \text{(Eq. 6.51c)}$$

This is applied in the discussion of vapor power cycles in **Chapter 8**.



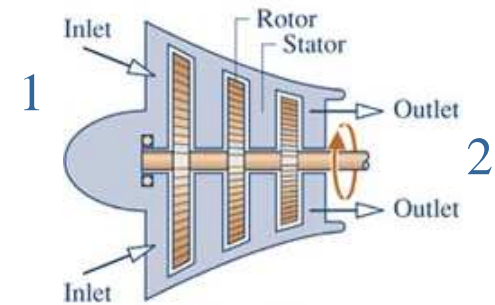
# Heat Transfer and Work in Internally Reversible Steady-State Flow Processes

- ▶ As shown by the figure, when the states visited by a unit mass passing from inlet to exit without internal irreversibilities are described by a curve on a *p-v* diagram, the *magnitude* of  $\int v dp$  is shown by the area *behind* the curve.



# Heat Transfer and Work in Internally Reversible Steady-State Flow Processes

**Example:** A compressor operates at steady state with natural gas entering at  $p_1, v_1$ . The gas undergoes a polytropic process described by  $pv = \text{constant}$  and exits at a higher pressure,  $p_2$ .



**(a)** Ignoring kinetic and potential energy effects, evaluate the work per unit of mass flowing.

**(b)** If internal irreversibilities were present, would the magnitude of the work per unit of mass flowing be less than, the same as, or greater than determined in part **(a)**?

# Heat Transfer and Work in Internally Reversible Steady-State Flow Processes

(a) With  $pv = \text{constant}$ , Eq. 6.51b gives

$$\begin{aligned} \left( \frac{\dot{W}_{cv}}{\dot{m}} \right)_{\text{int rev}} &= - \int_1^2 v dp = -\text{constant} \int_1^2 \frac{dp}{p} \\ &= -(p_1 v_1) \ln(p_2/p_1) \quad (\text{polytropic, } n = 1) \end{aligned}$$

The **minus sign** indicates that the compressor requires a **work input**.

(b) Left for class discussion.