

# **Chapter 4**

## **Control Volume Analysis Using Energy**

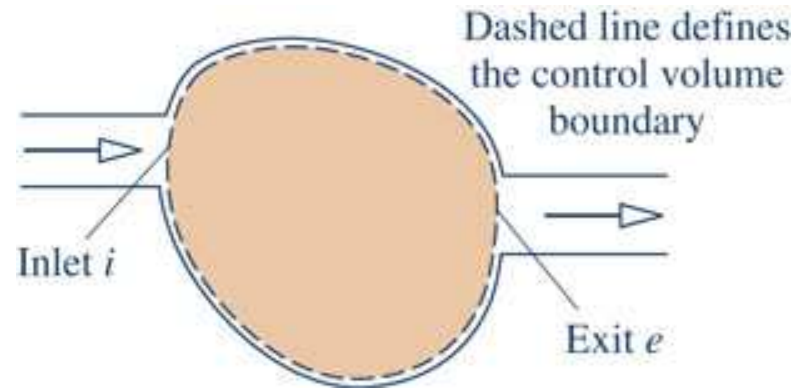
# Learning Outcomes

- ▶ Demonstrate understanding of key concepts related to control volume analysis including distinguishing between steady-state and transient analysis, distinguishing between mass flow rate and volumetric flow rate, and the meanings of one-dimensional flow and flow work.
- ▶ Apply mass and energy balances to control volumes.

# Learning Outcomes, cont.

- ▶ Develop appropriate **engineering models for control volumes**, with particular attention to analyzing components commonly encountered in engineering practice such as **nozzles, diffusers, turbines, compressors, heat exchangers, throttling devices, and integrated systems** that incorporate two or more components.
- ▶ Use **property data** in **control volume analysis** appropriately.

# Mass Rate Balance



$$\left[ \begin{array}{l} \text{time } \textit{rate of change} \text{ of} \\ \text{mass contained within the} \\ \text{control volume } \textit{at time } t \end{array} \right] = \left[ \begin{array}{l} \text{time } \textit{rate of flow} \text{ of} \\ \text{mass } \textit{in} \text{ across} \\ \text{inlet } \textit{i at time } t \end{array} \right] - \left[ \begin{array}{l} \text{time } \textit{rate of flow} \\ \text{of mass } \textit{out} \text{ across} \\ \text{exit } \textit{e at time } t \end{array} \right]$$

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

**(Eq. 4.1)**

# Mass Rate Balance

In practice there may be several locations on the boundary through which mass enters or exits. **Multiple inlets** and **exits** are accounted for by **introducing summations**:

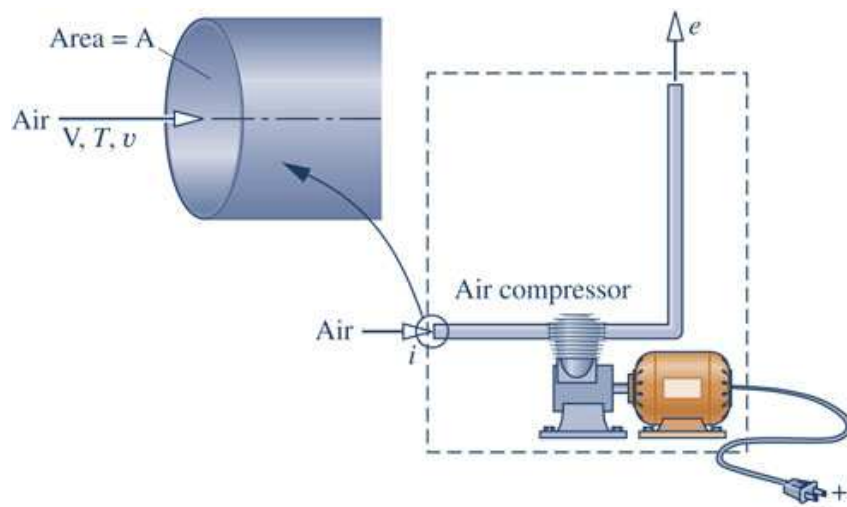
$$\frac{dm_{\text{CV}}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e \quad (\text{Eq. 4.2})$$

Eq. 4.2 is the **mass rate balance** for control volumes with **several inlets and exits**.

# Mass Flow Rate

## (One-Dimensional Flow)

- ▶ **Flow is normal** to the boundary at locations where mass enters or exits the control volume.
- ▶ **All** intensive properties are **uniform with position** over each inlet or exit area ( $A$ ) through which matter flows.



$$\dot{m} = \frac{AV}{v} \quad (\text{Eq. 4.4b})$$

where

$V$  is velocity

$v$  is specific volume

# Mass Rate Balance (Steady-State Form)

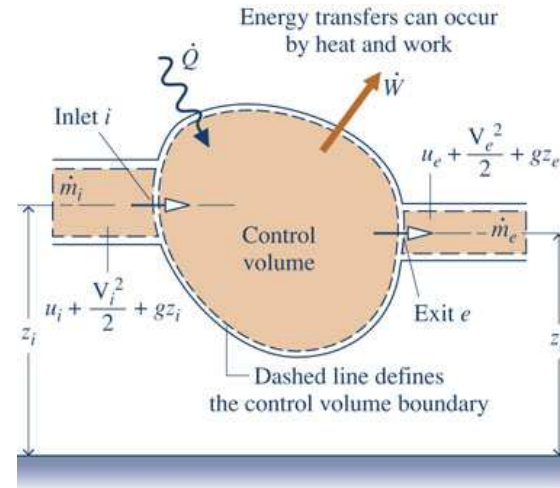
- ▶ **Steady-state**: all properties are unchanging in time.
- ▶ For steady-state control volume,  $dm_{cv}/dt = 0$ .

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

(Eq. 4.6)

(mass rate in)      (mass rate out)

# Energy Rate Balance



$$\left[ \begin{array}{l} \text{time } \textit{rate of change} \\ \text{of the energy} \\ \text{contained within} \\ \text{the control volume} \\ \textit{at time } t \end{array} \right] = \left[ \begin{array}{l} \textit{net rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred in} \\ \text{by heat transfer} \\ \textit{at time } t \end{array} \right] - \left[ \begin{array}{l} \textit{net rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred out} \\ \text{by work } \textit{at} \\ \textit{time } t \end{array} \right] + \left[ \begin{array}{l} \textit{net rate} \text{ of energy} \\ \text{transfer } \textit{into} \text{ the} \\ \text{control volume} \\ \text{accompanying} \\ \text{mass flow} \end{array} \right]$$

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \dot{m}_i \left( u_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left( u_e + \frac{V_e^2}{2} + gz_e \right) \quad \text{(Eq. 4.9)}$$



# Evaluating Work for a Control Volume

The expression for work is

$$\dot{W} = \dot{W}_{\text{cv}} + \dot{m}_e(p_e v_e) - \dot{m}_i(p_i v_i) \quad \text{(Eq. 4.12)}$$

where

- ▶  $\dot{W}_{\text{cv}}$  accounts for work associated with **rotating shafts, displacement of the boundary, and electrical effects.**
- ▶  $\dot{m}_e(p_e v_e)$  is the **flow work** at exit  $e$ .
- ▶  $\dot{m}_i(p_i v_i)$  is the **flow work** at inlet  $i$ .

# Control Volume Energy Rate Balance (One-Dimensional Flow Form)

Using Eq. 4.12 in Eq. 4.9

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i \left( \underline{u_i + p_i v_i} + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left( \underline{u_e + p_e v_e} + \frac{V_e^2}{2} + gz_e \right)$$

(Eq. 4.13)

For convenience substitute enthalpy,  $h = u + pv$

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)$$

(Eq. 4.14)

# Control Volume Energy Rate Balance (One-Dimensional Flow Form)

In practice there may be several locations on the boundary through which mass enters or exits. **Multiple inlets** and **exits** are accounted for by **introducing summations**:

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)$$

(Eq. 4.15)

Eq. 4.15 is the **accounting balance** for the energy of the control volume.

# Control Volume Energy Rate Balance (Steady-State Form)

- ▶ **Steady-state**: all properties are unchanging in time.
- ▶ For steady-state control volume,  $dE_{cv}/dt = 0$ .

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)$$

(Eq. 4.18)

# Control Volume Energy Rate Balance (Steady-State Form, One-Inlet, One-Exit)

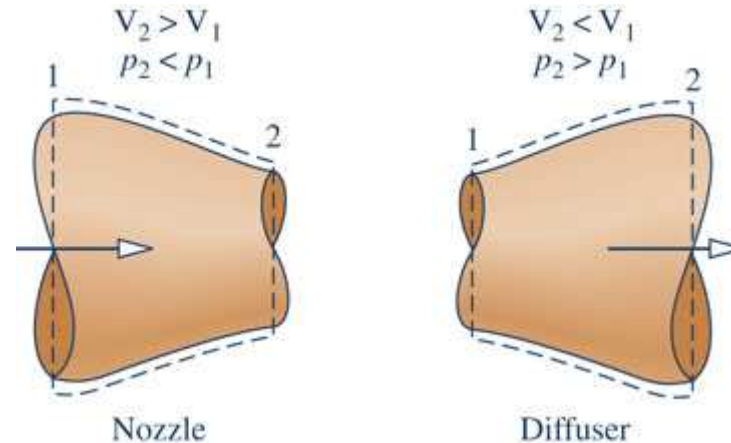
- ▶ Many important applications involve **one-inlet**, **one-exit** control volumes at **steady state**.
- ▶ The mass rate balance reduces to  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ .

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \quad \left( \text{Eq. 4.20a} \right)$$

or dividing by mass flow rate

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \quad \left( \text{Eq. 4.20b} \right)$$

# Nozzles and Diffusers



- ▶ **Nozzle**: a flow passage of varying cross-sectional area in which the **velocity** of a gas or liquid **increases** in the direction of flow.
- ▶ **Diffuser**: a flow passage of varying cross-sectional area in which the **velocity** of a gas or liquid **decreases** in the direction of flow.

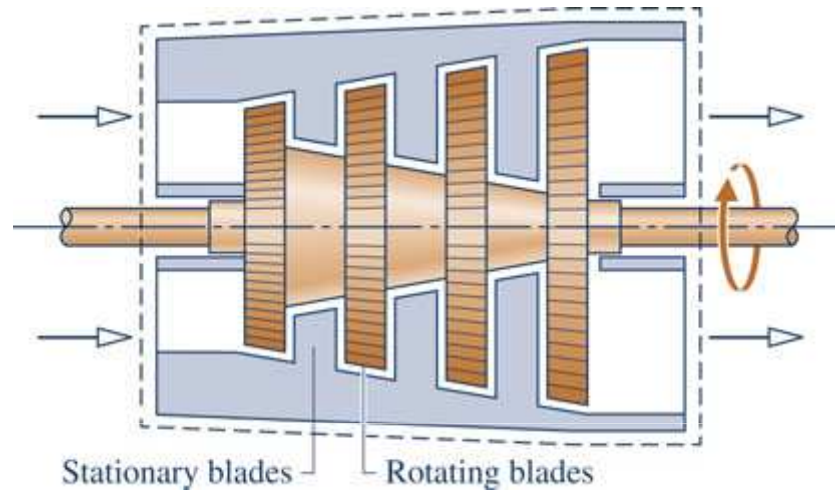
# Nozzle and Diffuser Modeling

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \quad \left( \text{Eq. 4.20a} \right)$$

- ▶  $\dot{W}_{cv} = 0$ .
- ▶ If the change in potential energy from inlet to exit is negligible,  $g(z_1 - z_2)$  drops out.
- ▶ If the heat transfer with surroundings is negligible,  $\dot{Q}_{cv}$  drops out.

$$0 = (h_1 - h_2) + \left( \frac{V_1^2 - V_2^2}{2} \right) \quad \text{(Eq. 4.21)}$$

# Turbines



- ▶ **Turbine:** a device in which **power is developed** as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate.



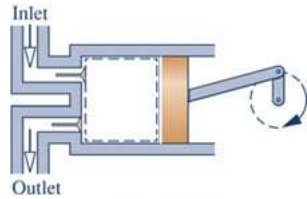
# Turbine Modeling

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \quad \left( \text{Eq. 4.20a} \right)$$

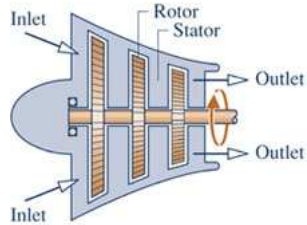
- ▶ If the change in kinetic energy of flowing matter is negligible,  $\frac{1}{2}(V_1^2 - V_2^2)$  drops out.
- ▶ If the change in potential energy of flowing matter is negligible,  $g(z_1 - z_2)$  drops out.
- ▶ If the heat transfer with surroundings is negligible,  $\dot{Q}_{cv}$  drops out.

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

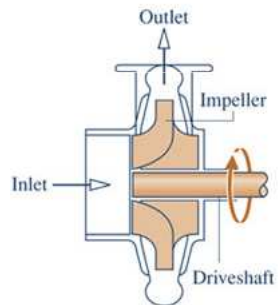
# Compressors and Pumps



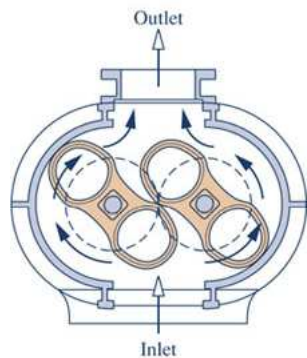
(a) Reciprocating



(b) Axial flow



(c) Centrifugal



(d) Roots type

- ▶ **Compressors and Pumps:** devices in which work is done on the substance flowing through them to change the state of the substance, typically to increase the pressure and/or elevation.
- ▶ **Compressor:** substance is gas
- ▶ **Pump:** substance is liquid

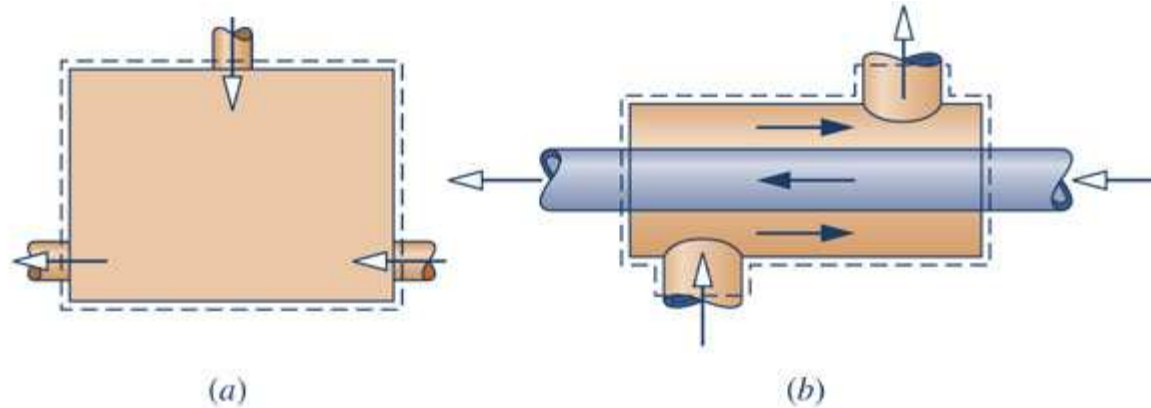
# Compressor and Pump Modeling

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \quad \left( \text{Eq. 4.20a} \right)$$

- ▶ If the change in kinetic energy of flowing matter is negligible,  $\frac{1}{2}(V_1^2 - V_2^2)$  drops out.
- ▶ If the change in potential energy of flowing matter is negligible,  $g(z_1 - z_2)$  drops out.
- ▶ If the heat transfer with surroundings is negligible,  $\dot{Q}_{cv}$  drops out.

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

# Heat Exchangers



- ▶ **Direct contact:** A mixing chamber in which hot and cold streams are *mixed directly*.
- ▶ **Tube-within-a-tube counterflow:** A gas or liquid stream is *separated* from another gas or liquid by a wall through which energy is conducted. Heat transfer occurs from the hot stream to the cold stream as the streams flow in opposite directions.

# Heat Exchanger Modeling

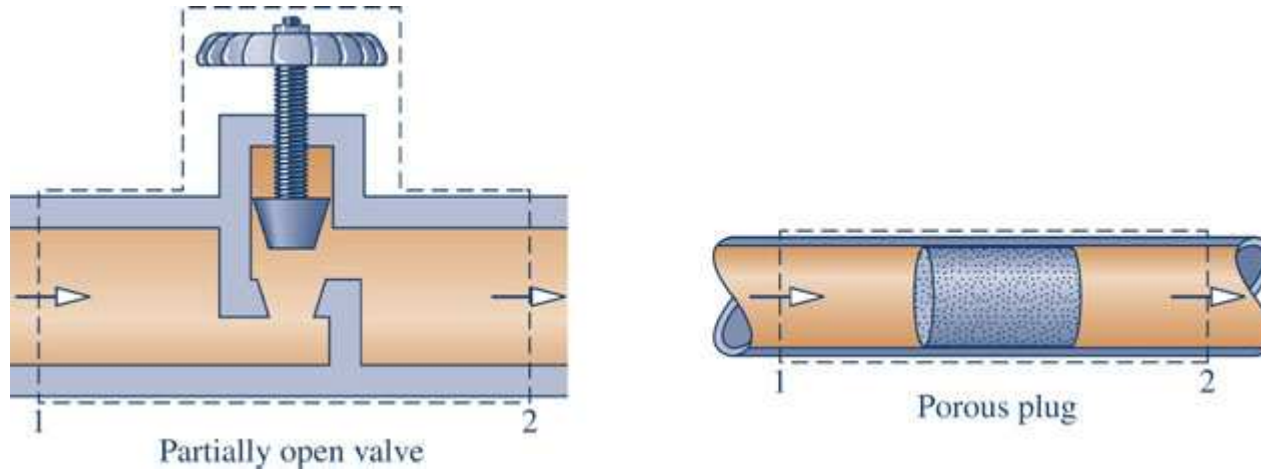
$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \sum_i \dot{m}_i \left( \cancel{h_i} + \cancel{\frac{V_i^2}{2}} + \cancel{gz_i} \right) - \sum_e \dot{m}_e \left( \cancel{h_e} + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e} \right)$$

(Eq. 4.18)

- ▶  $\dot{W}_{cv} = 0$ .
- ▶ If the kinetic energies of the flowing streams are negligible,  $\dot{m}_i(V_i^2/2)$  and  $\dot{m}_e(V_e^2/2)$  drop out.
- ▶ If the potential energies of the flowing streams are negligible,  $\dot{m}_i gz_i$  and  $\dot{m}_e gz_e$  drop out.
- ▶ If the heat transfer with surroundings is negligible,  $\dot{Q}_{cv}$  drops out.

$$0 = \sum_i \dot{m}_i h_i - \sum_e \dot{m}_e h_e$$

# Throttling Devices



- ▶ **Throttling Device**: a device that achieves a significant **reduction in pressure** by introducing a restriction into a line through which a gas or liquid flows. Means to introduce the restriction include a partially opened valve or a porous plug.

# Throttling Device Modeling

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2) \right] \quad \left( \text{Eq. 4.20a} \right)$$

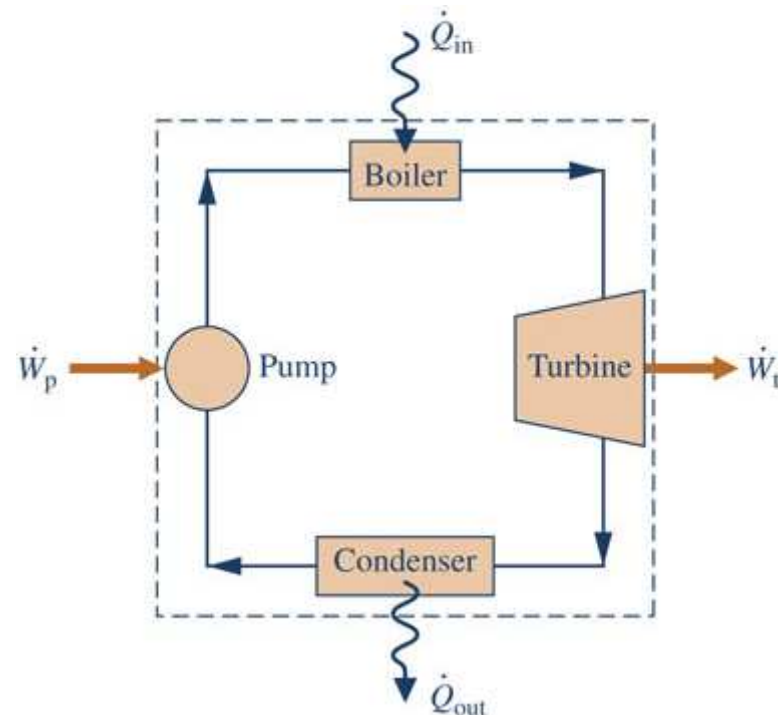
- ▶  $\dot{W}_{cv} = 0$ .
- ▶ If the change in kinetic energy of flowing matter upstream and downstream of the restriction is negligible,  $\frac{1}{2}(V_1^2 - V_2^2)$  drops out.
- ▶ If the change in potential energy of flowing matter is negligible,  $g(z_1 - z_2)$  drops out.
- ▶ If the heat transfer with surroundings is negligible,  $\dot{Q}_{cv}$  drops out.

$$h_2 = h_1$$

**(Eq. 4.22)**

# System Integration

- ▶ Engineers creatively combine components to achieve some overall objective, subject to constraints such as minimum total cost. This engineering activity is called **system integration**.
- ▶ The simple vapor power plant of **Fig 4.16** provides an illustration.





# The Mass Balance (Transient Analysis)

- ▶ **Transient**: state changes with time.
- ▶ **Integrate** mass rate balance (**Eq. 4.2**) from time **0** to a final time ***t***.

$$\int_0^t \left( \frac{dm_{cv}}{dt} \right) dt = \int_0^t \left( \sum_i \dot{m}_i \right) dt - \int_0^t \left( \sum_e \dot{m}_e \right) dt$$

This becomes  $m_{cv}(t) - m_{cv}(0) = \sum_i m_i - \sum_e m_e$  (**Eq. 4.23**)

where

- $m_i$  is amount of **mass entering** the control volume **through inlet *i***, from time **0** to ***t***.
- $m_e$  is amount of **mass exiting** the control volume **through exit *e***, from time **0** to ***t***.

# The Energy Balance (Transient Analysis)

- **Integrate** energy rate balance (Eq. 4.15), ignoring the effects of kinetic and potential energy, from time **0** to a final time ***t***.

$$\int_0^t \left( \frac{dU_{cv}}{dt} \right) dt = \int_0^t \dot{Q}_{cv} dt - \int_0^t \dot{W}_{cv} dt + \int_0^t \left( \sum_i \dot{m}_i h_i \right) dt - \int_0^t \left( \sum_e \dot{m}_e h_e \right) dt$$

When the **specific enthalpies at inlets and exits are constant with time**, this becomes

$$U_{cv}(t) - U_{cv}(0) = Q_{cv} - W_{cv} + \sum_i m_i h_i - \sum_e m_e h_e \quad \text{(Eq. 4.25)}$$