Date and day

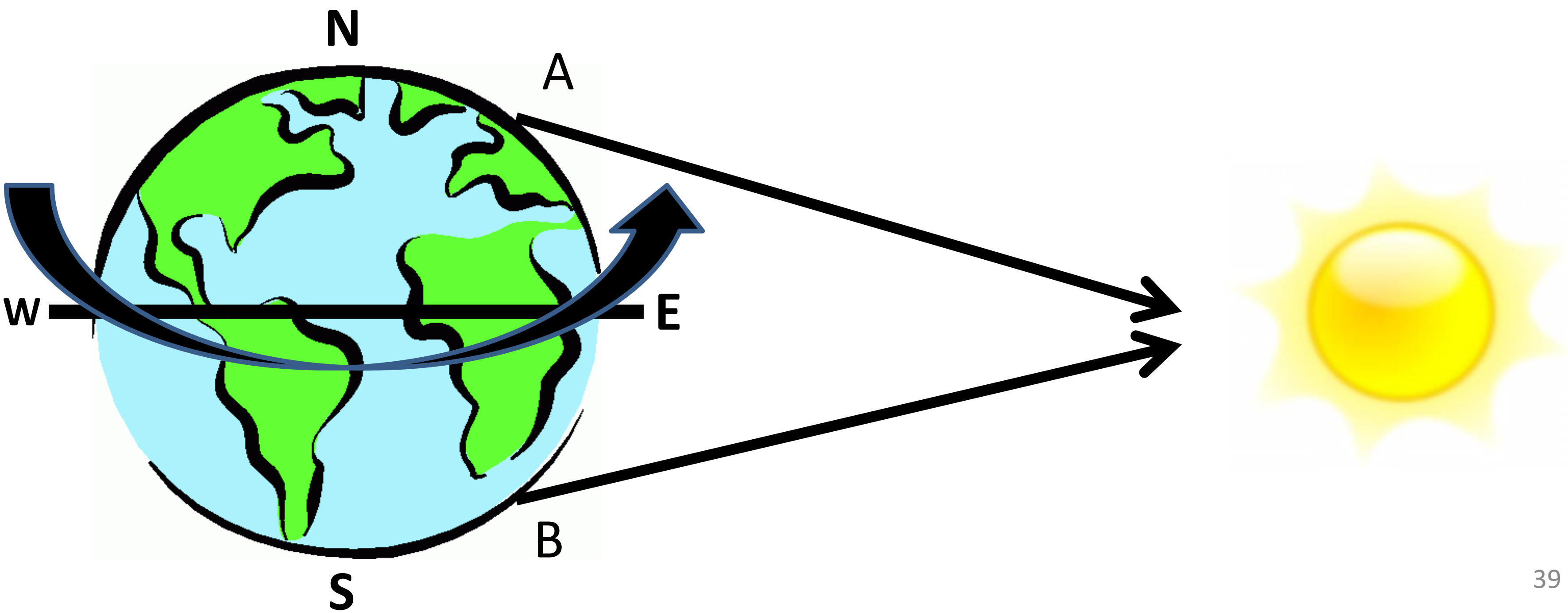
- Date is represented by month and ‘i’
- Day is represented by ‘n’

Month	**n^{th} day for i^{th} date**
January	i
February	31 + i
March	59 + i
…	…
December	334 + i

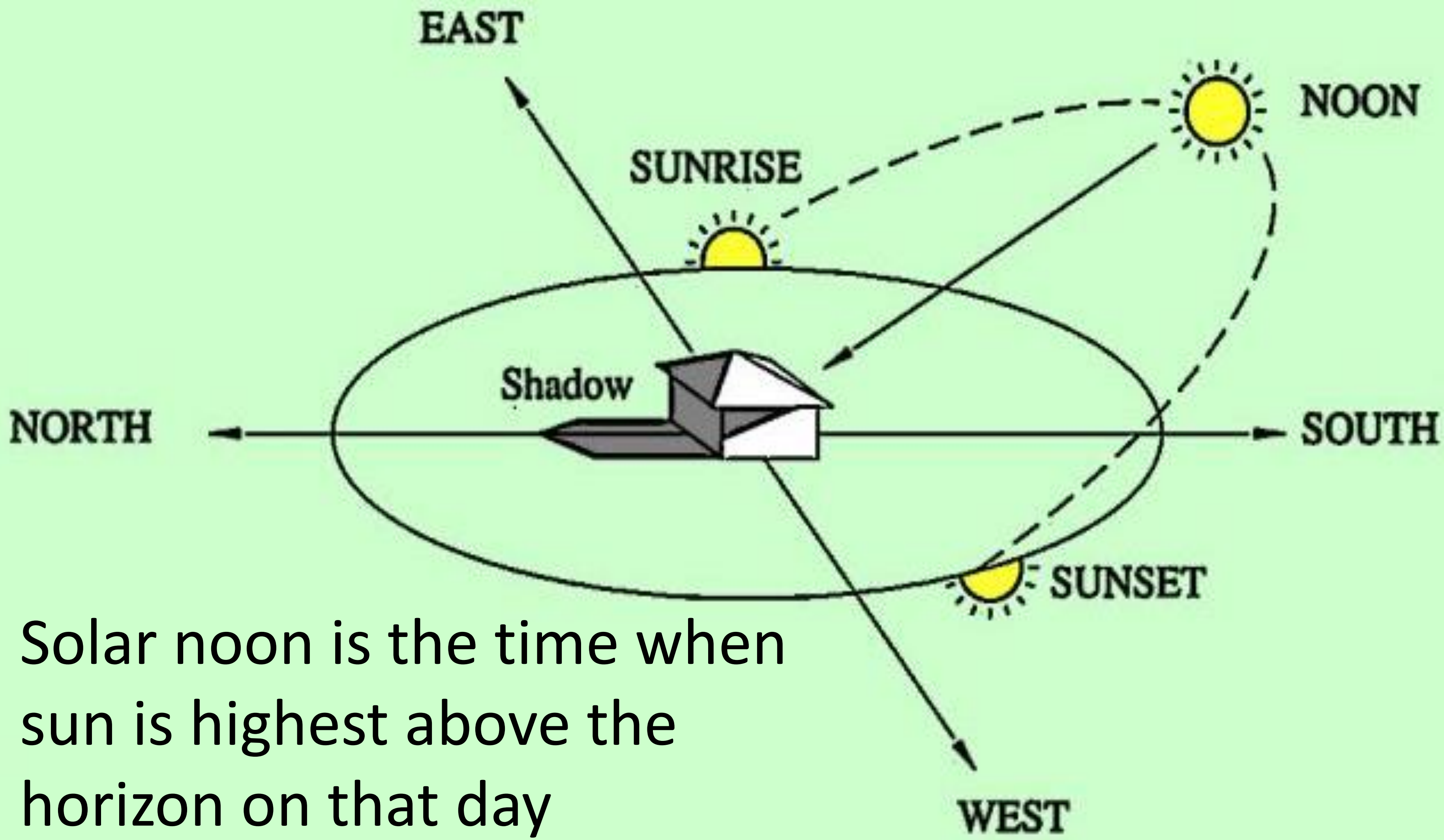
(See “Days in Year” in *Reference Information*)

Sun position from earth

- Sun rise in the east and set in the west
- "A" sees sun in south
- "B" sees sun in north



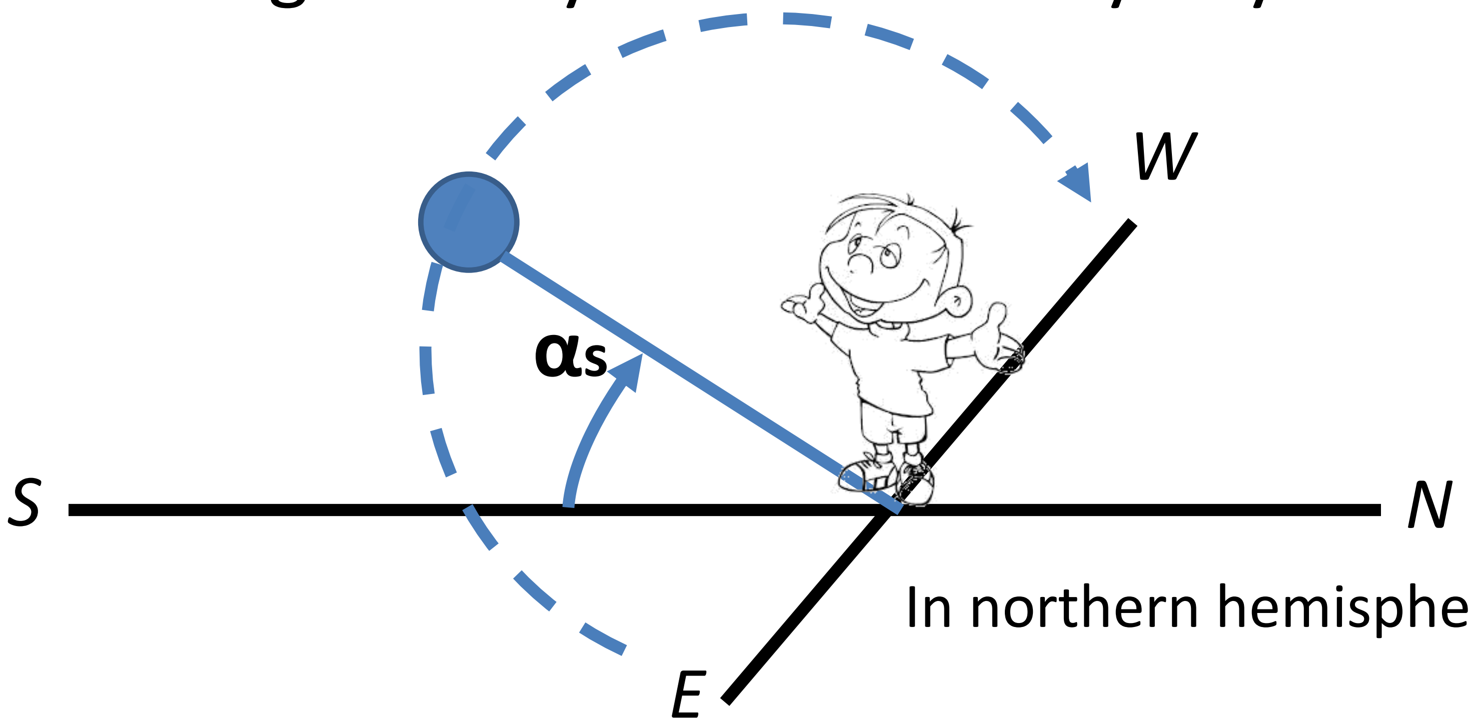
Solar noon



Solar noon is the time when sun is highest above the horizon on that day

Solar altitude angle

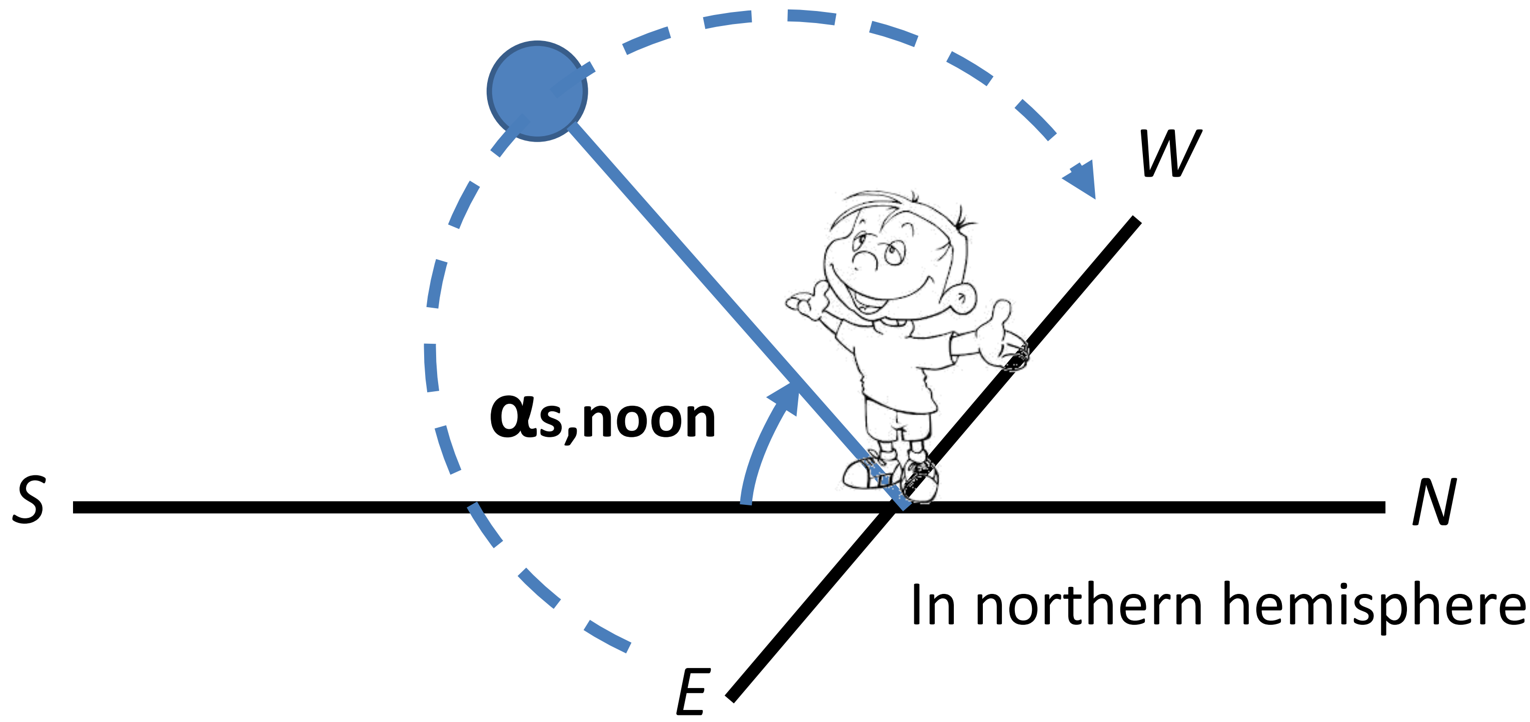
- Solar altitude angle (α_s) is the angle between horizontal and the line passing through sun
- It changes every hour and every day



In northern hemisphere

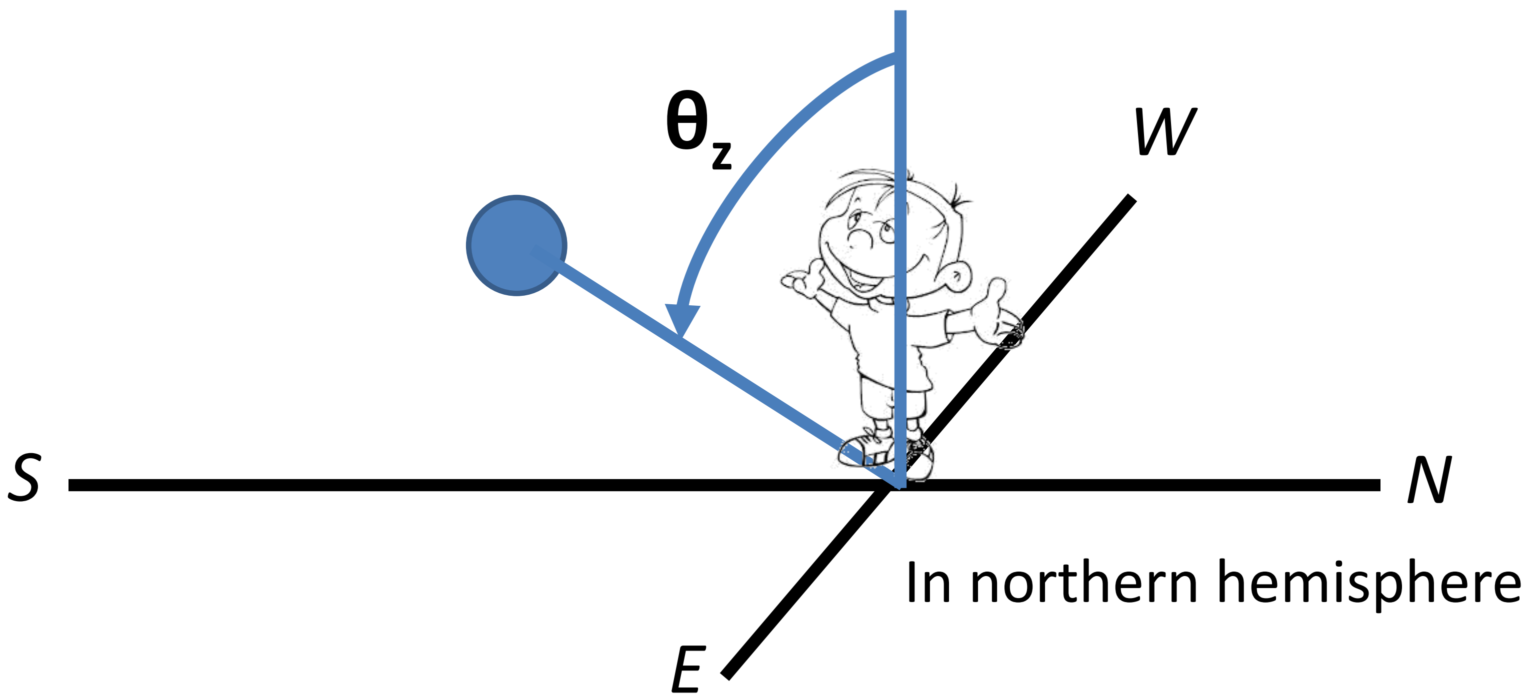
Solar altitude angle at noon

Solar altitude angle is maximum at "Noon" for a day, denoted by $\alpha_{s,noon}$



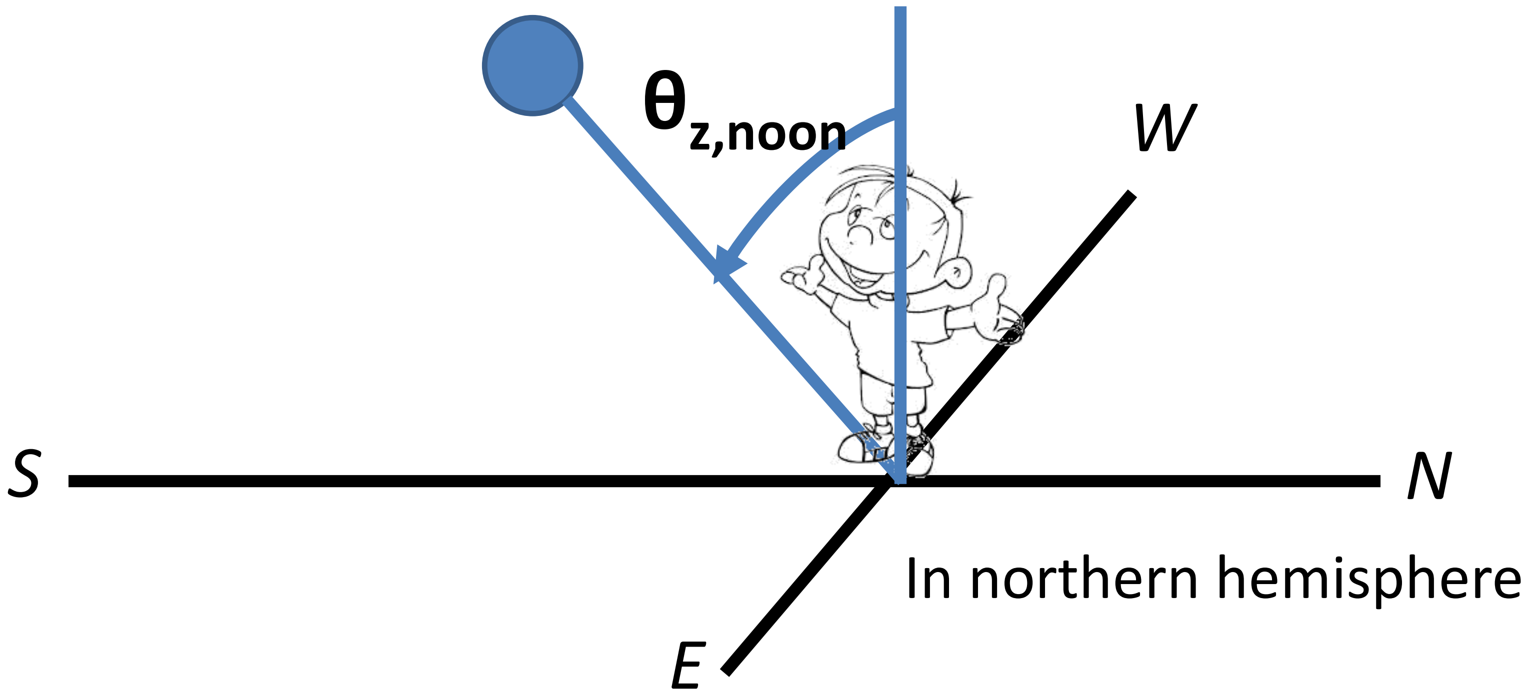
Zenith angle

- Zenith angle (θ_z) is the angle between vertical and the line passing through sun
- $\theta_z = 90 – \alpha_s$



Zenith angle at noon

- Zenith angle is minimum at "Noon" for a day, denoted by $\theta_{z,noon}$
- $\theta_{z,noon} = 90 - \alpha_{s,noon}$



Air mass

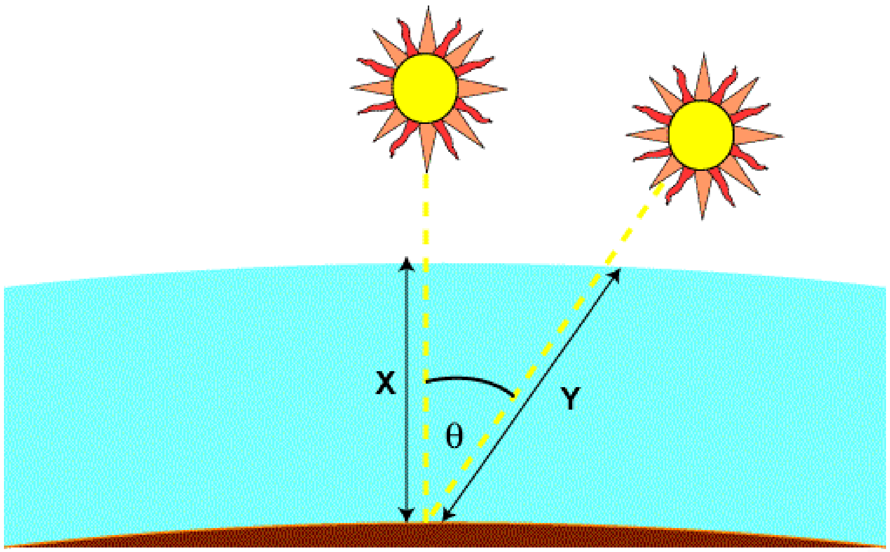
- Another representation of solar altitude/zenith angle.
- Air mass (A.M.) is the ratio of mass of atmosphere through which beam passes, to the mass it would pass through, if the sun were directly overhead.

$$A.M. = 1/\cos\theta_z$$

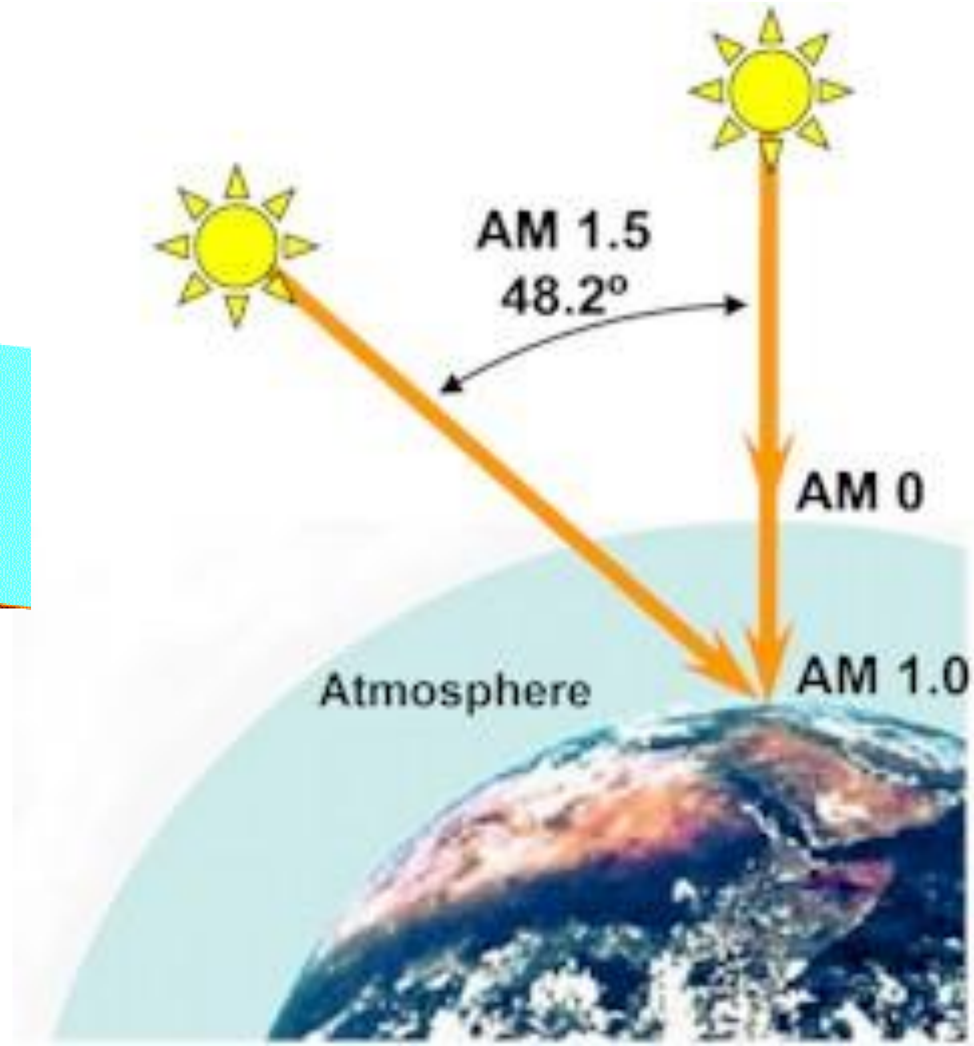
If A.M.=1 => θ_z=0° (Sun is directly overhead)

If A.M.=2 => θ_z=60° (Sun is away, a lot of mass of air is present between earth and sun)

Air mass

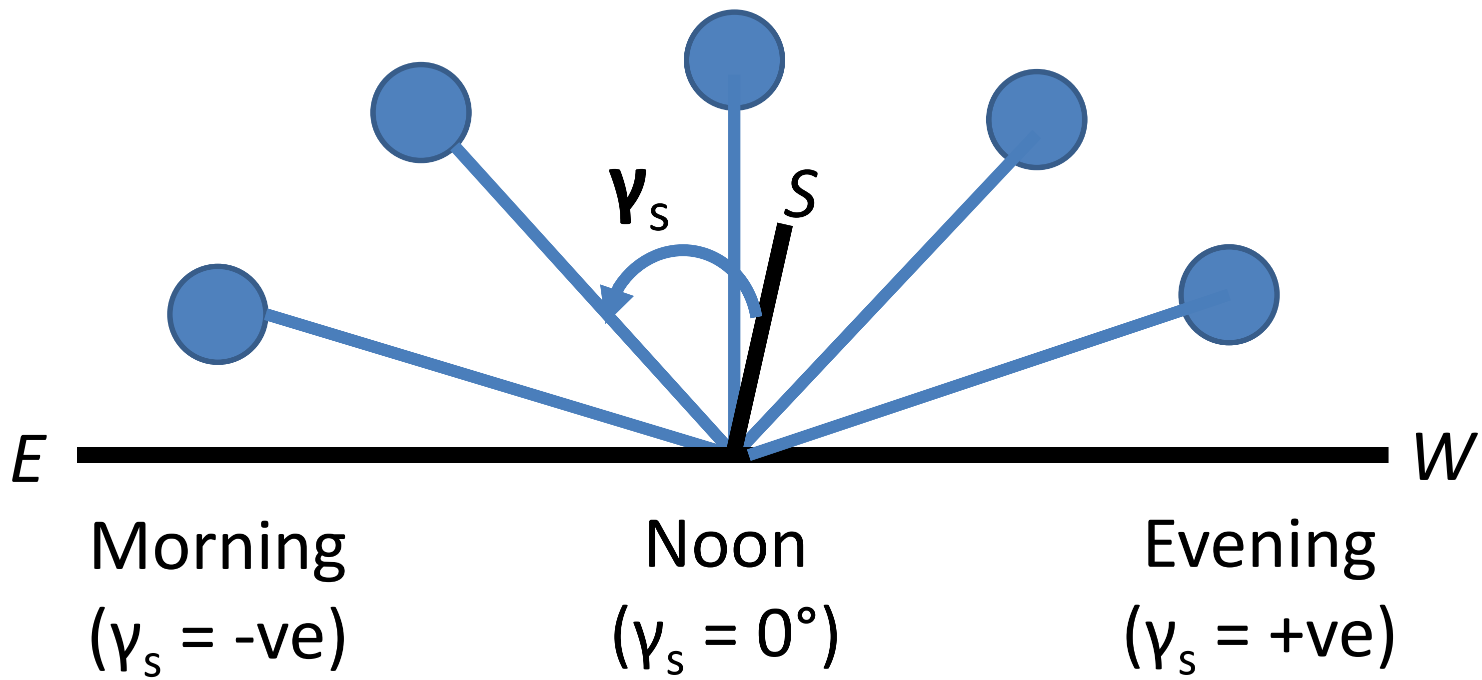


$$A.M. = 1/\cos\theta_z$$



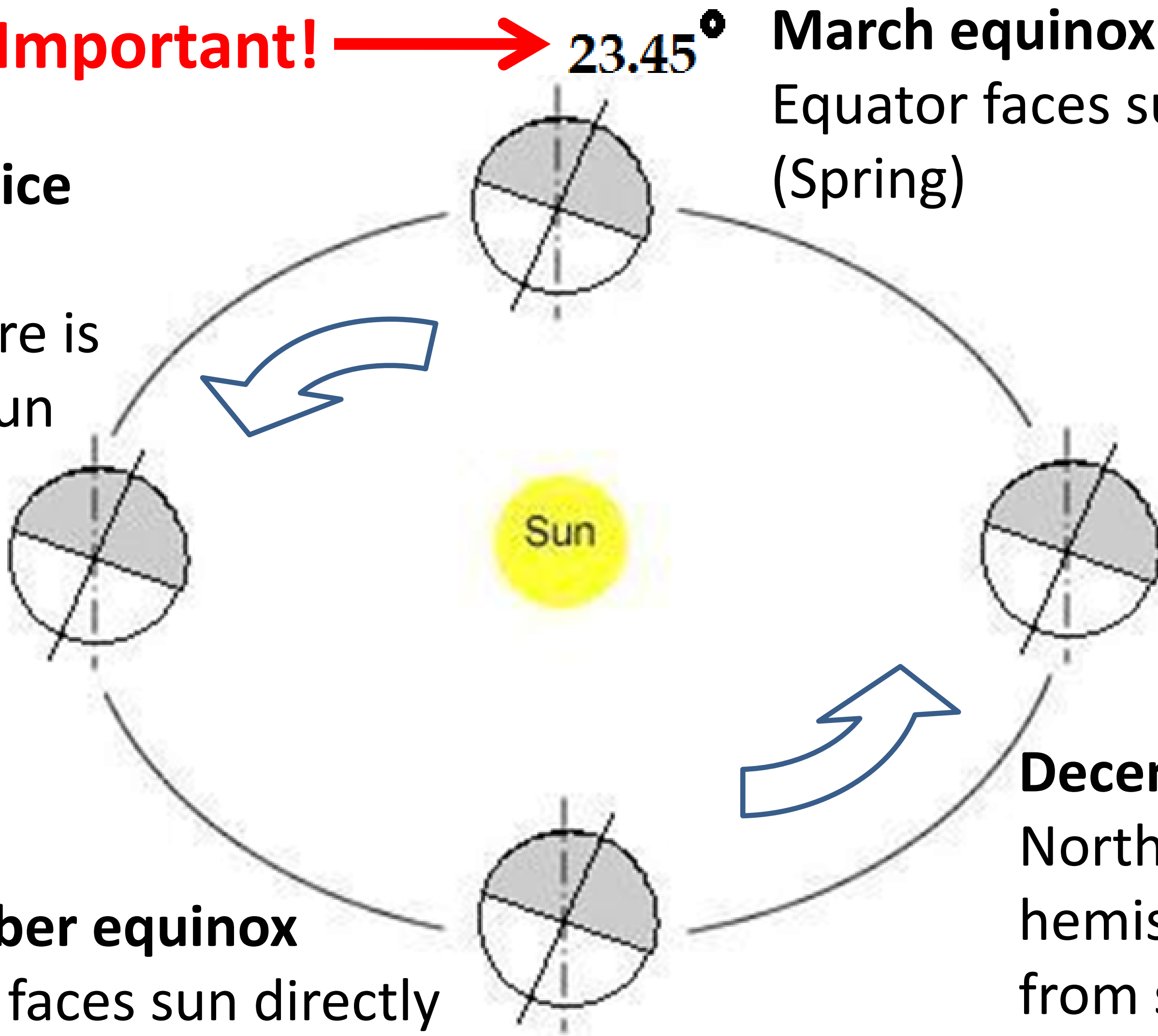
Solar azimuth angle

- In any hemisphere, solar azimuth angle (γ_s) is the angular displacement of sun from south
- It is 0° due south, -ve in east, +ve in west



Solar declination

Important! →



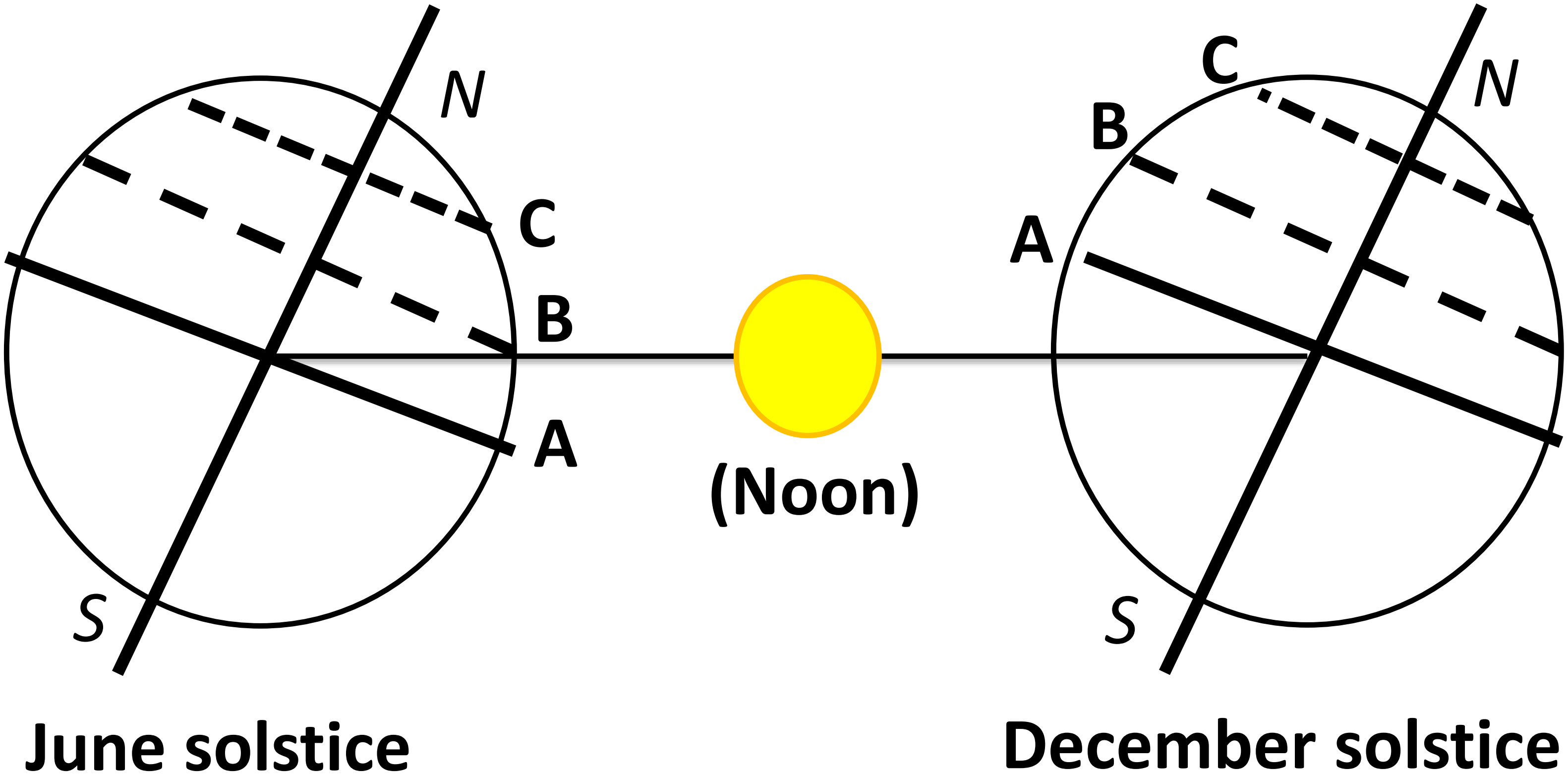
March equinox
Equator faces sun directly (Spring)

June solstice
Northern hemisphere is towards sun (Summer)

September equinox
Equator faces sun directly (Autumn)

December solstice
Northern hemisphere is away from sun (Winter)

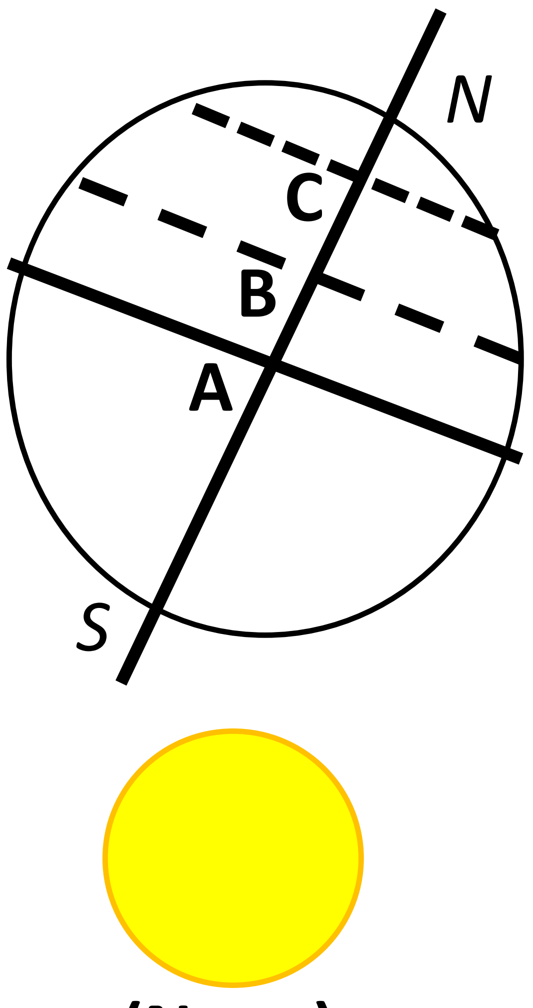
Solar declination (at solstice)



A sees sun in north.
B sees sun overhead.
C sees sun in south.

A sees sun in south.
B sees sun in *more* south.
C sees sun in *much more* south.

Solar declination (at equinox)



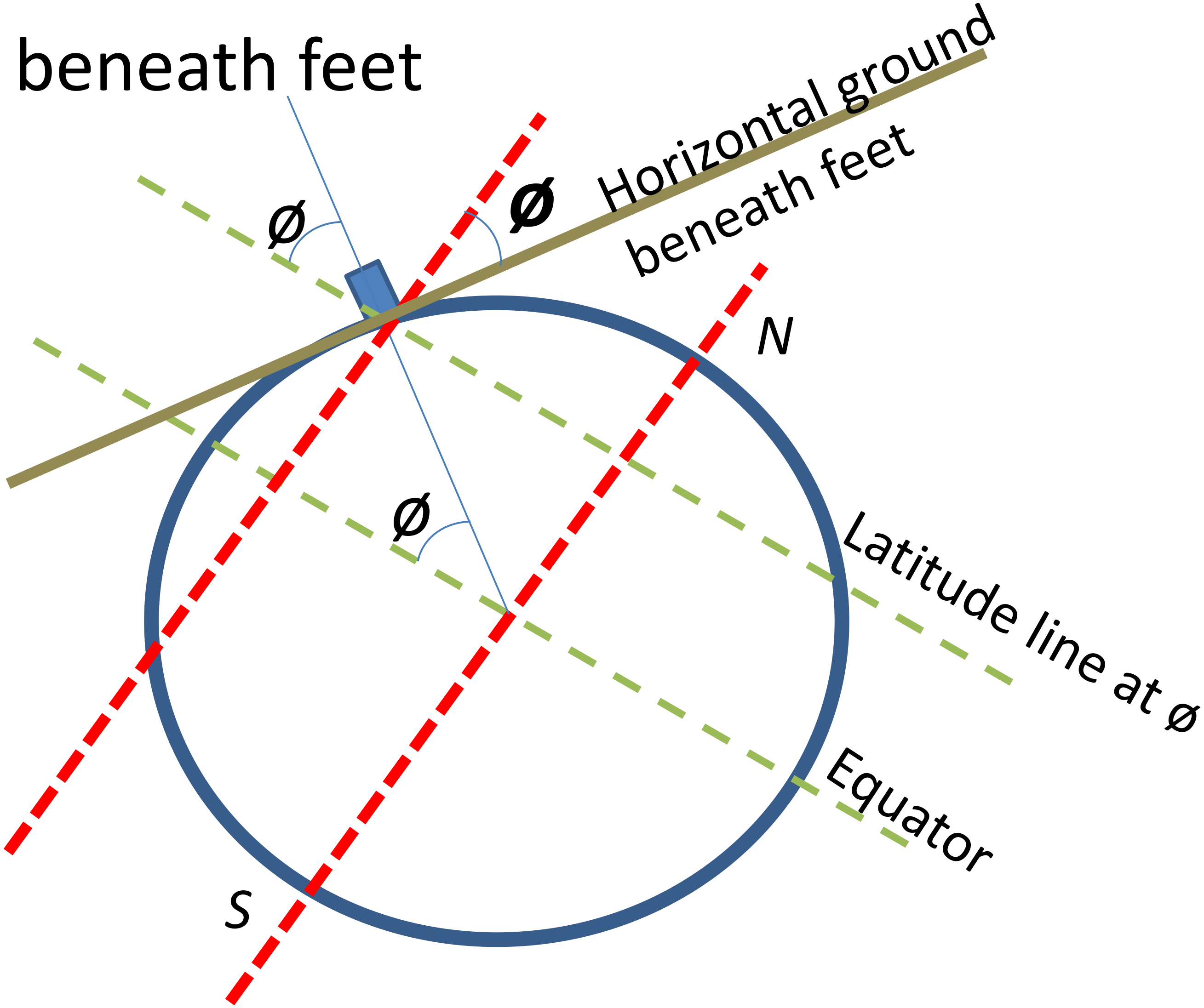
March equinox

A sees sun directly overhead
B sees sun in *more* south
C sees sun in *much more* south

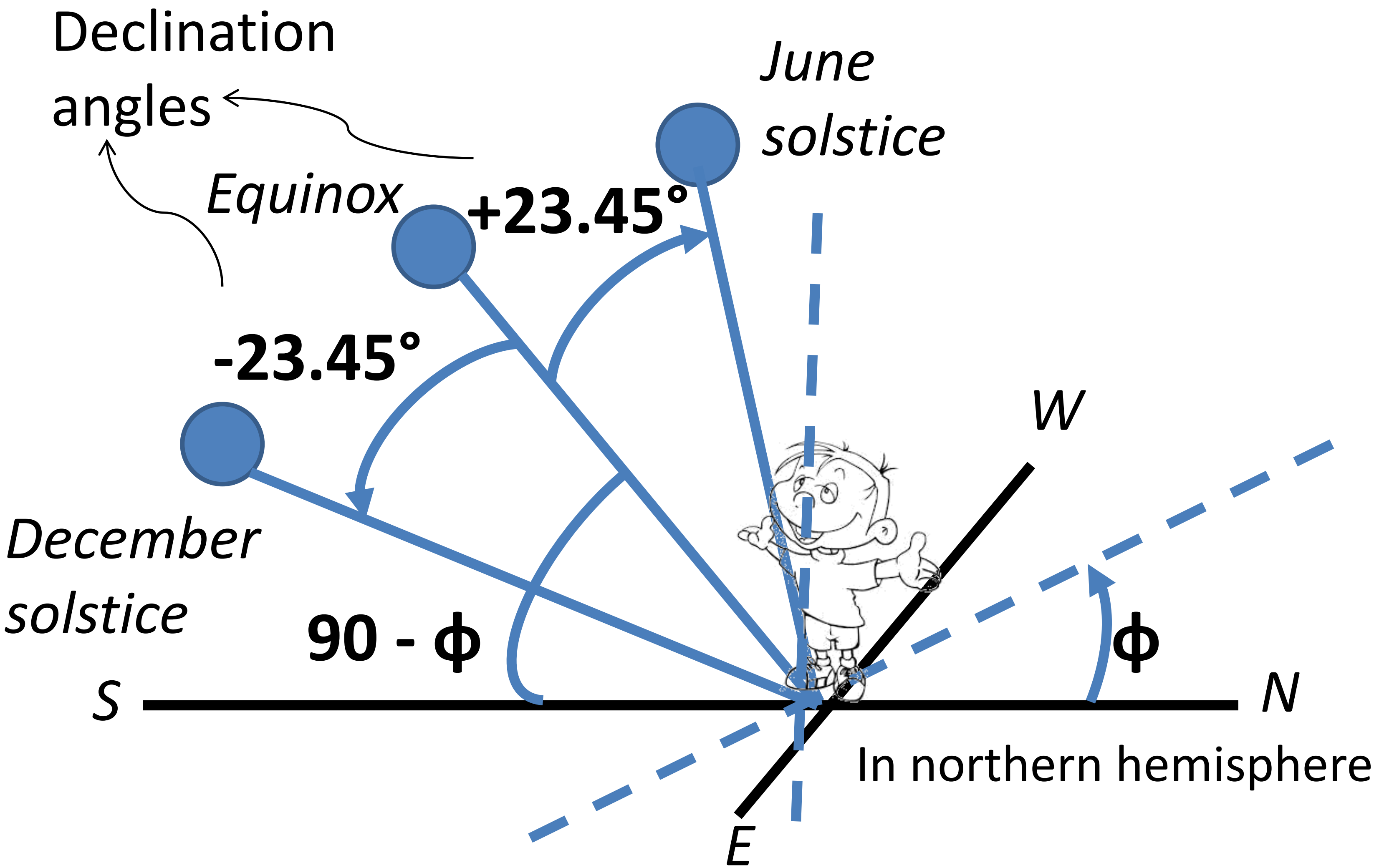
Same situation happen during September equinox.

Solar declination

Latitude from frame of reference of horizontal ground beneath feet



Solar declination



Note: Altitude depends upon latitude but declination is independent.

Solar declination

- For any day in year, solar declination (δ) can be calculated as:

$$\delta = 23.45 \sin\left(360\left(\frac{284+n}{365}\right)\right)$$

Where, n = numberth day of year

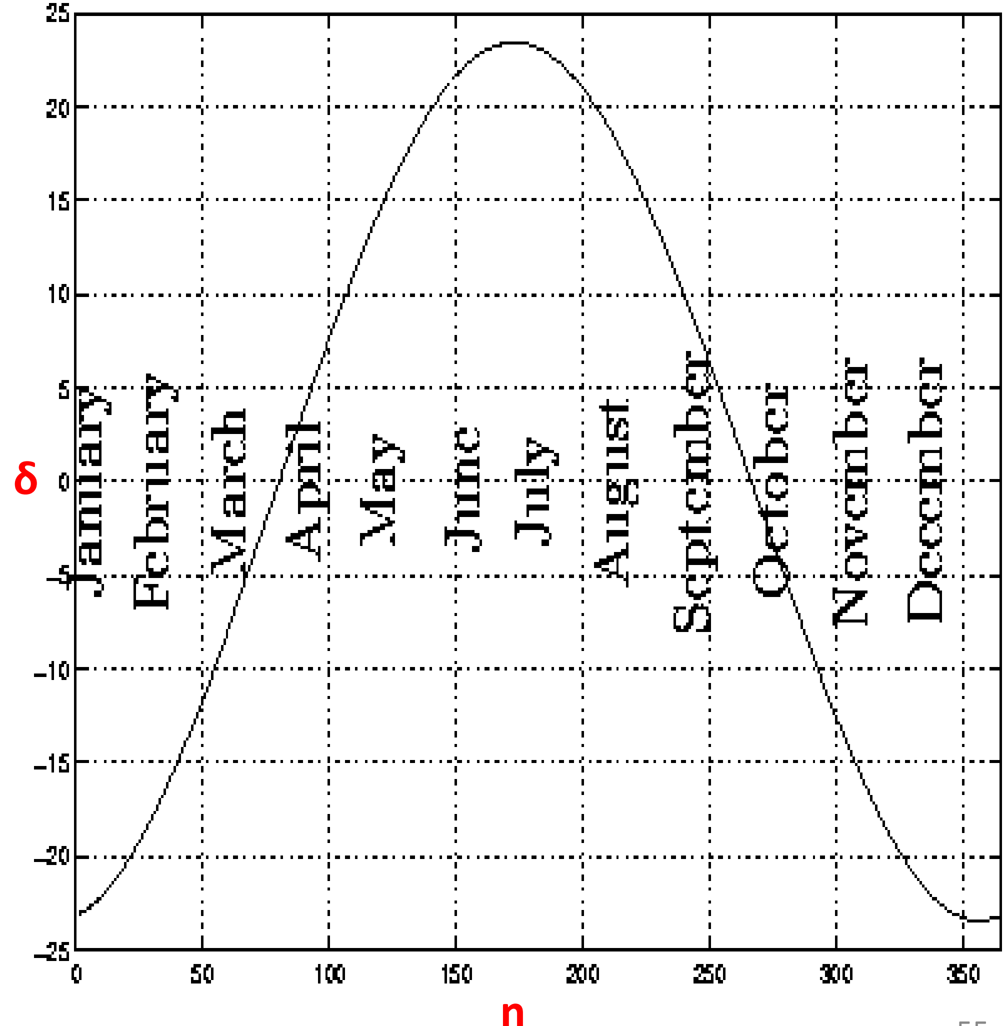
(See "Days in Year" in *Reference Information*)

- Maximum: 23.45 °, Minimum: -23.45°
- Solar declination angle represents "day"
- It is independent of time and location!

Solar declination

Days to Remember	δ
March, 21	0°
June, 21	+23.45°
September, 21	0°
December, 21	-23.45°

Can you prove this?



Solar altitude and zenith at noon

- As solar declination (δ) is the function of day (n) in year, therefore, solar altitude at noon can be calculated as:

$$\alpha_{s,noon} = 90 - \phi + \delta$$

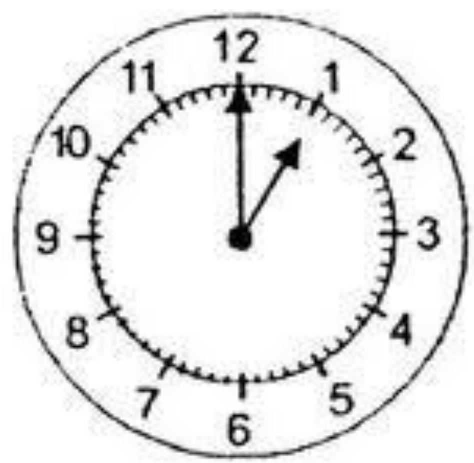
- Similarly zenith angle at noon can be calculated as:

$$\theta_{z,noon} = 90 - \alpha_{s,noon} = 90 - (90 - \phi + \delta) = \phi - \delta$$

Solar time

- The time in your clock (local time) is not same as "solar time"
- It is always "Noon" at 12:00pm *solar time*

Solar time "Noon"



Local time (in your clock)

Solar time

The difference between solar time (ST) and local time (LT) can be calculated as:

$$ST - LT = E - \frac{4 \times (SL - LL)}{60}$$

Where,

ST: Solar time (in 24 hours format)

LT: Local time (in 24 hours format)

SL: Standard longitude (depends upon GMT)

LL: Local longitude (+ve for east, -ve for west)

E: Equation of time (in hours)

Try: http://www.powerfromthesun.net/soltimecalc.html

Solar time

- Standard longitude (SL) can be calculated as:

$$SL = (GMT \times 15)$$

- Where GMT is Greenwich Mean Time, roughly:

If LL > 0 (Eastward):

$$GMT = ceil(LL/15)$$

If LL < 0 (Westward):

$$GMT = -floor(|LL|/15)$$

- GMT for Karachi is 5, GMT for Tehran is 3.5.
- It is recommended to find GMT from standard database e.g. *http://wwp.greenwichmeantime.com/*

Solar time

- The term Equation of time (E) is because of earth's tilt and orbit eccentricity.
- It can be calculated as:

$$E = \frac{229.2}{60} \times \begin{pmatrix} 0.000075 \\ +0.001868 \cos B \\ -0.032077 \sin B \\ -0.014615 \cos 2B \\ -0.04089 \sin 2B \end{pmatrix}$$

Where,

$$B = (n-1)360/365$$

Hour angle

- Hour angle (ω) is another representation of solar time
- It can be calculated as:

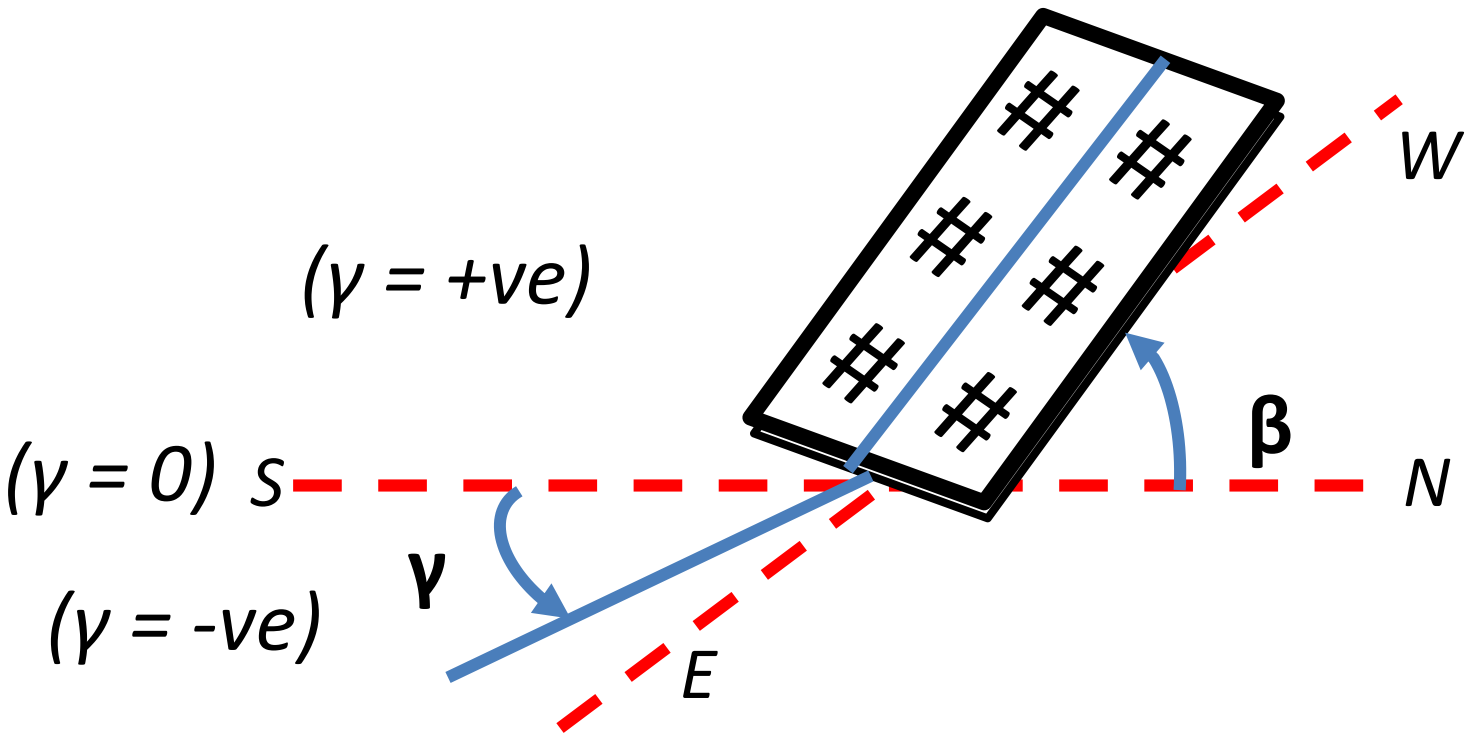
$$\omega = (ST - 12) \times 15$$

(-ve before solar noon, +ve after solar noon)

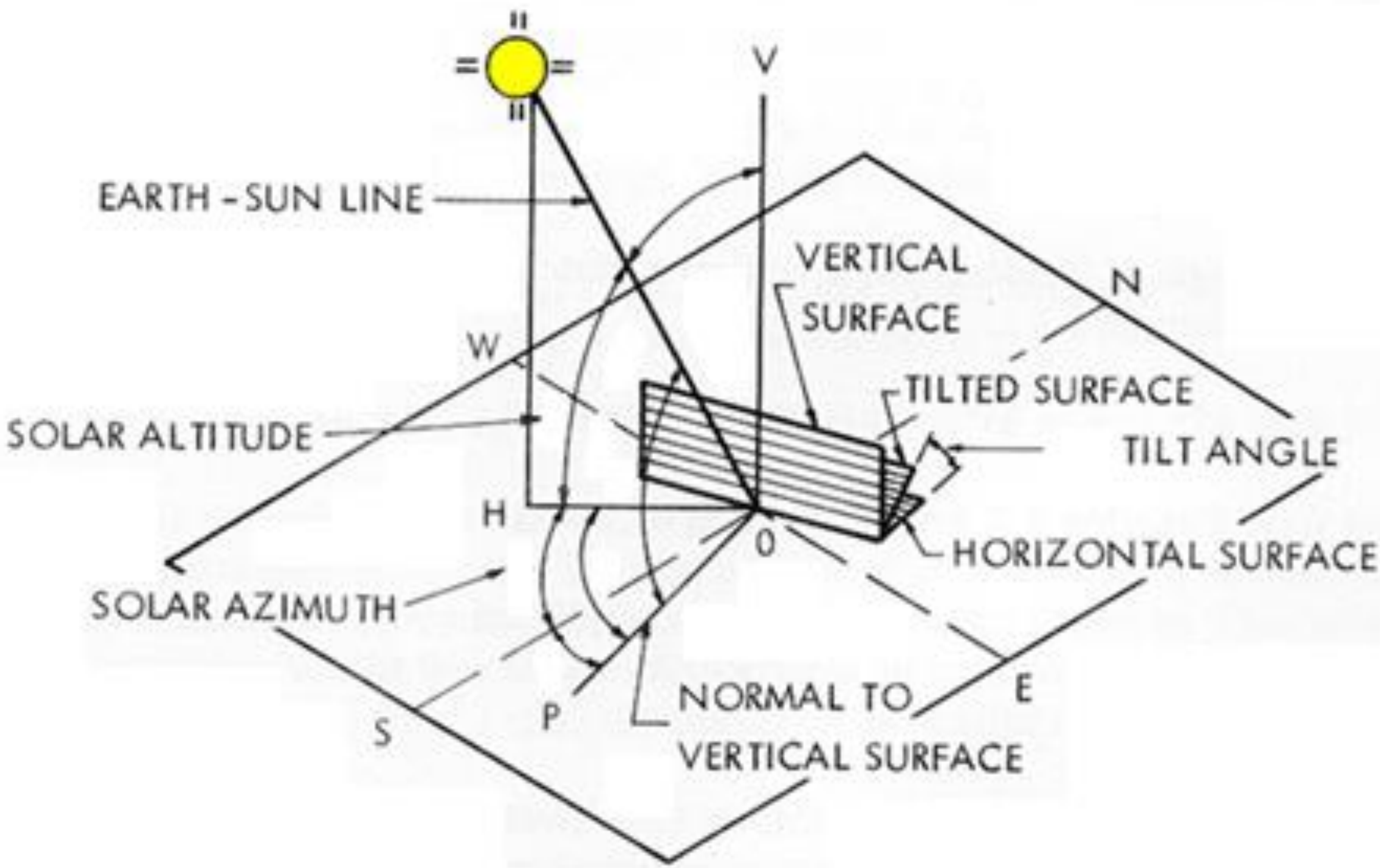
11:00am ω = -15°	12:00pm ω = 0°	01:00pm ω = +15°

A plane at earth's surface

- Tilt, pitch or slope angle: β (in degrees)
- Surface azimuth or orientation: γ (in degrees, 0° due south, -ve in east, +ve in west)



Summary of solar angles



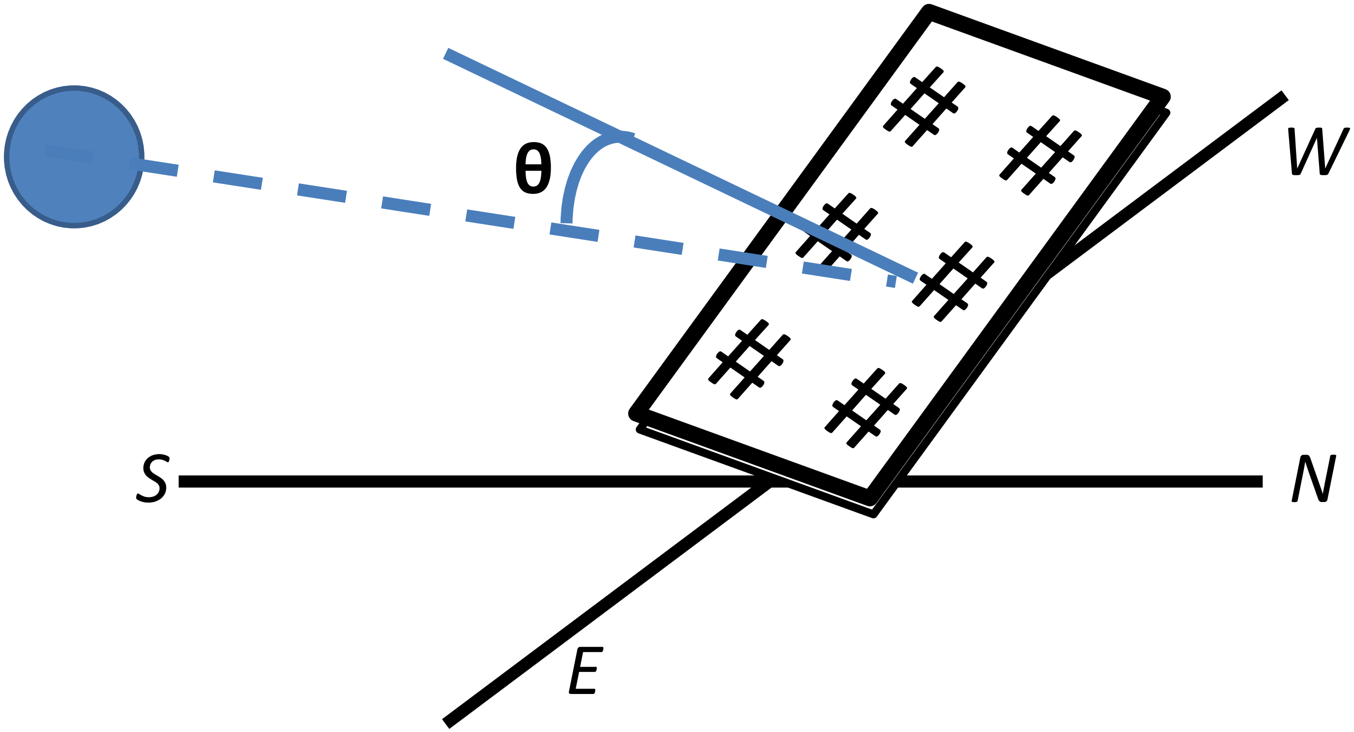
Can you write symbols of different solar angles shown in this diagram?

Interpretation of solar angles

Angle		Interpretation	Set#
Latitude	ϕ	Site location	1
Declination	δ	Day (Sun position)	2
Hour angle	ω	Time (Sun position)	
Solar altitude	α_s	Sun direction (Sun position)	3
Zenith angle	θ_z	Sun direction (Sun position)	
Solar azimuth	γ_s	Sun direction (Sun position)	
Tilt angle	β	Plane direction	4
Surface azimuth	γ	Plane direction	

Angle of incidence

Angle of incidence (θ) is the angle between normal of plane and line which is meeting plane and passing through the sun



Angle of incidence

- Angle of incidence (θ) depends upon:
  - **Site location** (1): θ changes place to place
  - **Sun position** (2/3): θ changes in every instant of time and day
  - **Plane direction** (4): θ changes if plane is moved
- It is 0° for a plane directly facing sun and at this angle, maximum solar radiations are collected by plane.

Angle of incidence

If the sun position is known in terms of declination (day) and hour angle, angle of incidence (θ) can be calculated as:

$$\begin{aligned}\cos\theta &= \sin\delta\sin\emptyset\cos\beta - \sin\delta\cos\emptyset\sin\beta\cos\gamma \\ &+ \cos\delta\cos\emptyset\cos\beta\cos\omega \\ &+ \cos\delta\sin\emptyset\sin\beta\cos\gamma\cos\omega \\ &+ \cos\delta\sin\beta\sin\gamma\sin\omega\end{aligned}$$

(Set 1+2+4)

Angle of incidence

If the sun position is known in terms of sun direction (i.e. solar altitude/zenith and solar azimuth angles), angle of incidence (θ) can be calculated as:

$$\cos\theta = \cos\theta_z \cos\beta + \sin\theta_z \sin\beta \cos(\gamma_s - \gamma)$$

Remember, $\theta_z = 90 - \alpha_s$

Note: Solar altitude/zenith angle and solar azimuth angle depends upon location.

(Set 1+3+4)

Special cases for angle of incidence

- If the plane is laid horizontal *(β=0°)*
  - Equation is independent of γ *(rotate!)*
  - θ becomes θ_z because normal to the plane becomes vertical, hence:

$$\cos\theta_z = \cos\emptyset\cos\delta\cos\omega + \sin\emptyset\sin\delta$$

Remember, $\theta_z = 90 - \alpha_s$

Note: Solar altitude/zenith angle depends upon location, day and hour.

Solar altitude and azimuth angle

Solar altitude angle (α_s) can be calculated as:

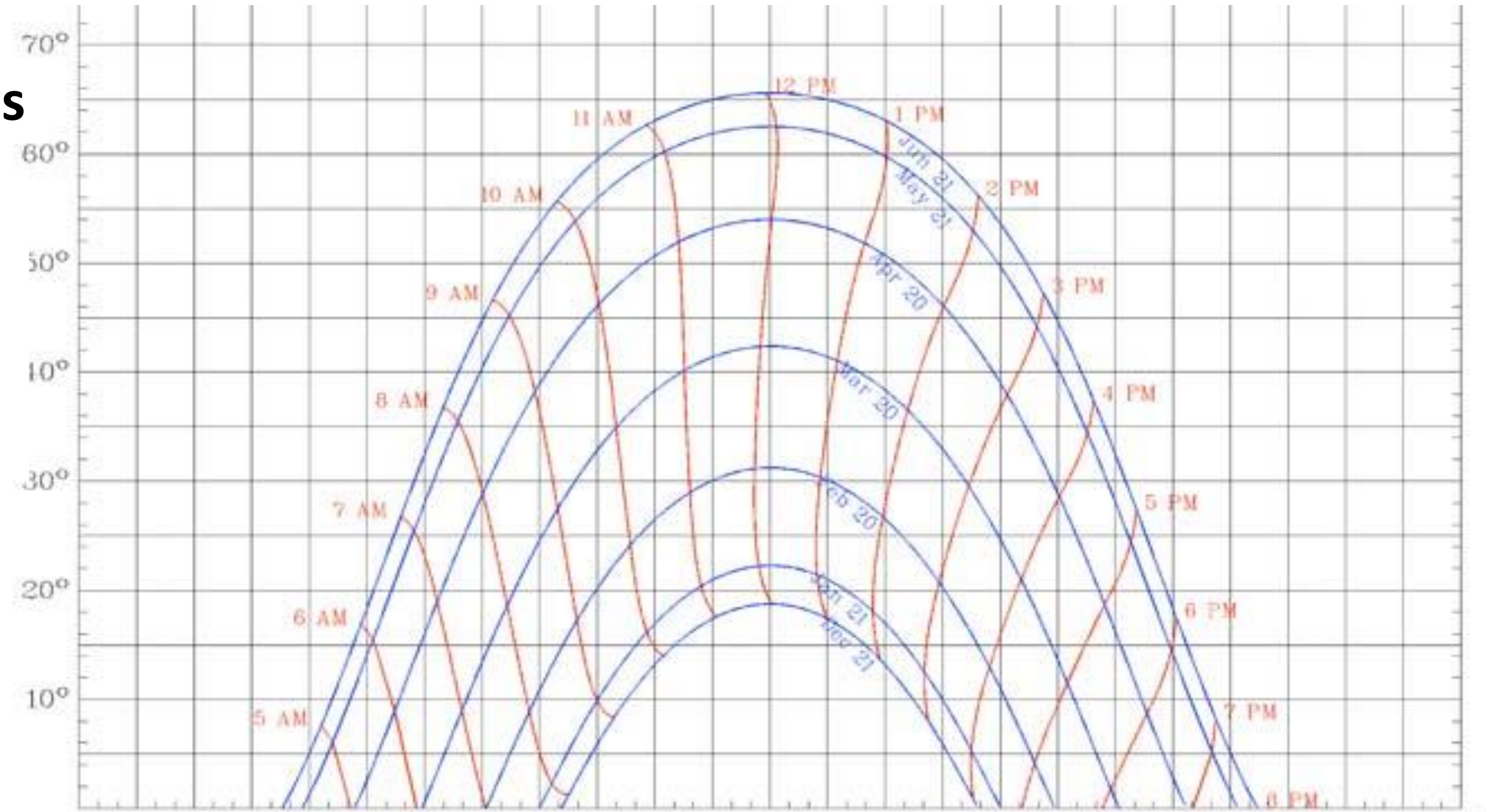
$$\sin\alpha_s = \cos\emptyset \cos\delta \cos\omega + \sin\emptyset \sin\delta$$

Solar azimuth angle (γ_s) can be calculated as:

$$\gamma_s = \text{sign}(\omega) \left| \cos^{-1}\left(\frac{\cos\theta_z \sin\emptyset - \sin\delta}{\sin\theta_z \cos\emptyset}\right) \right|$$

Sun path diagram or sun charts

α_s



-150° -120° -90° - 60° -30° 0° 30° 60° 90° 120° 150°

γ_s

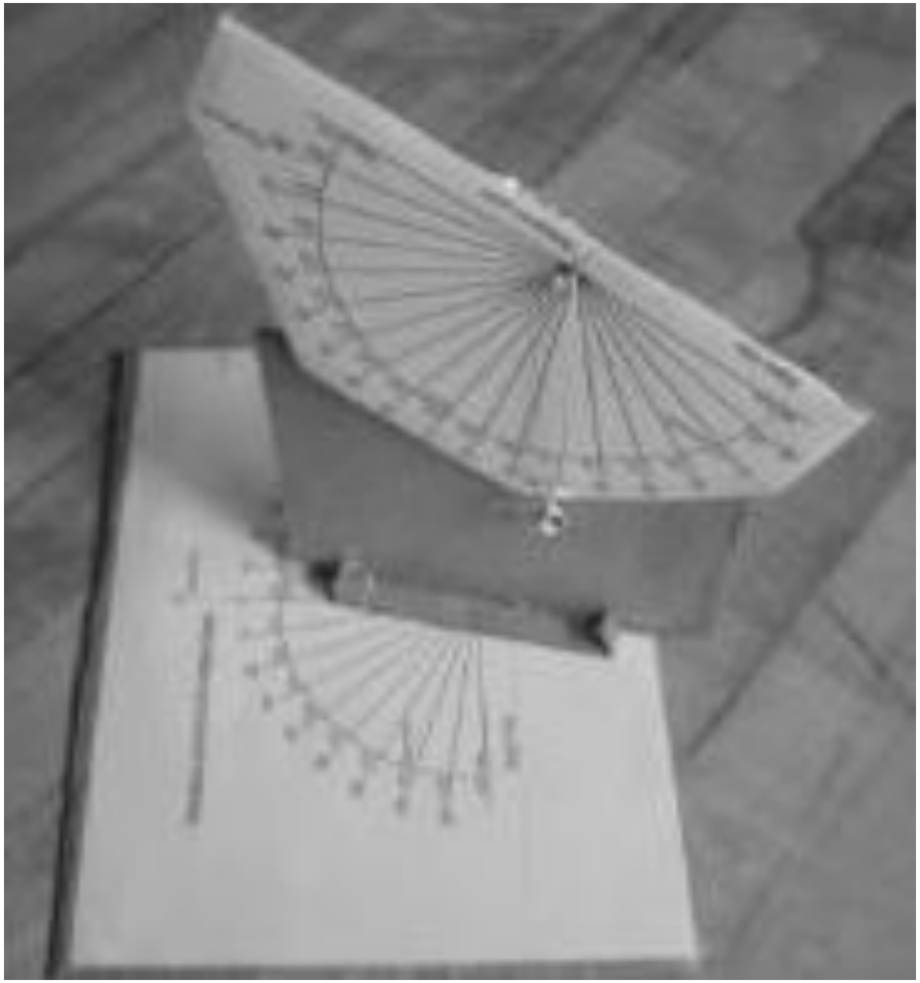
Note: These diagrams are different for different latitudes.

Shadow analysis (objects at distance)

- Shadow analysis for objects at distance (e.g. trees, buildings, poles etc.) is done to find:
  - Those moments (hours and days) in year when plane will not see sun.
  - Loss in total energy collection due to above.
- Mainly, following things are required:
  - Sun charts for site location
  - Inclinometer
  - Compass and information of M.D.

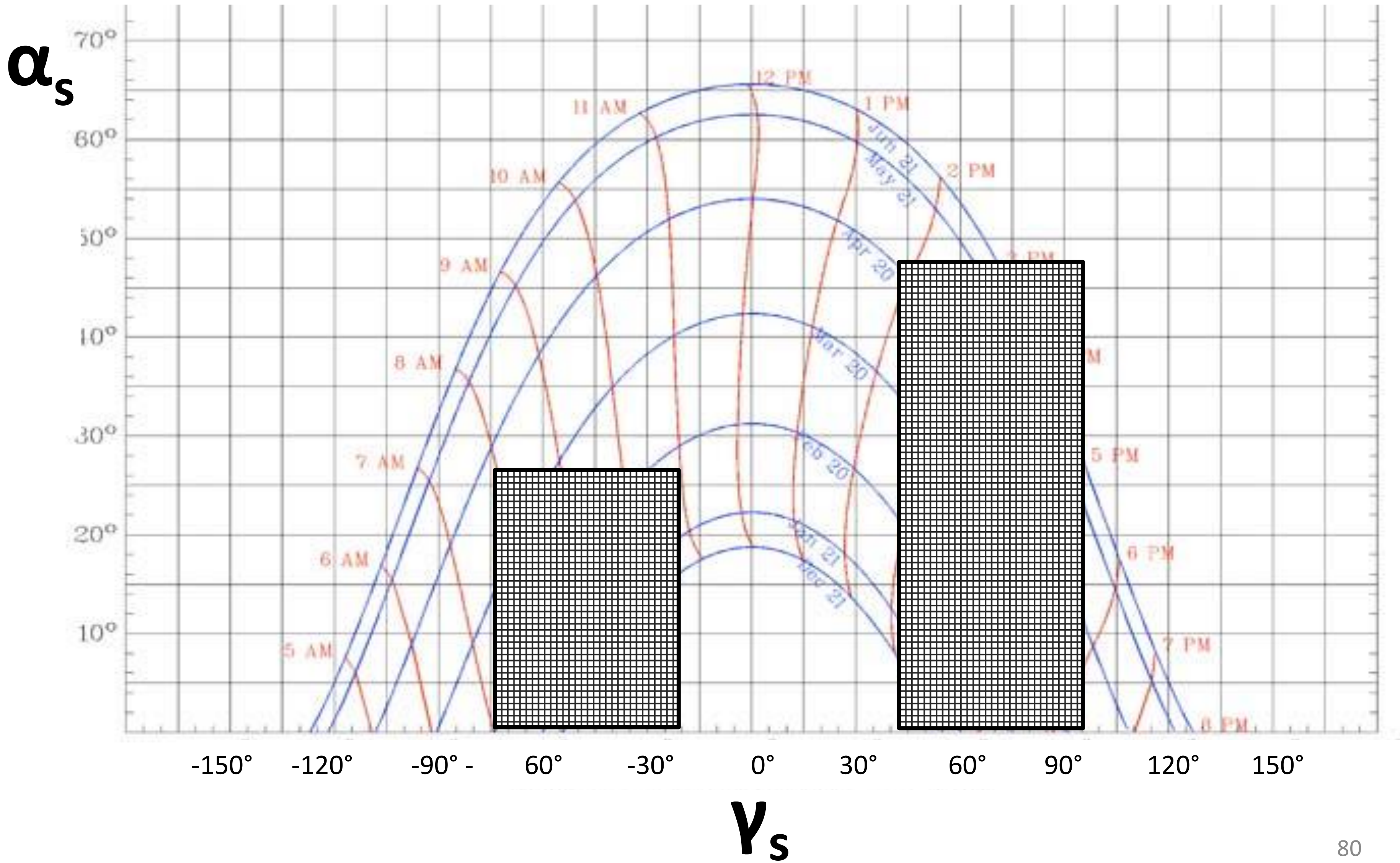
Inclinometer

A simple tool for finding azimuths and altitudes of objects



http://rimstar.org/renewnrg/solar_site_survey_shading_location.htm

Shadow analysis using sun charts



Sunset hour angle and daylight hours

- Sunset occurs when $\theta_z = 90°$ (or $\alpha_s = 0°$). Sunset hour angle (ω_s) can be calculated as:

$$\cos\omega_s = -\tan\emptyset\tan\delta$$

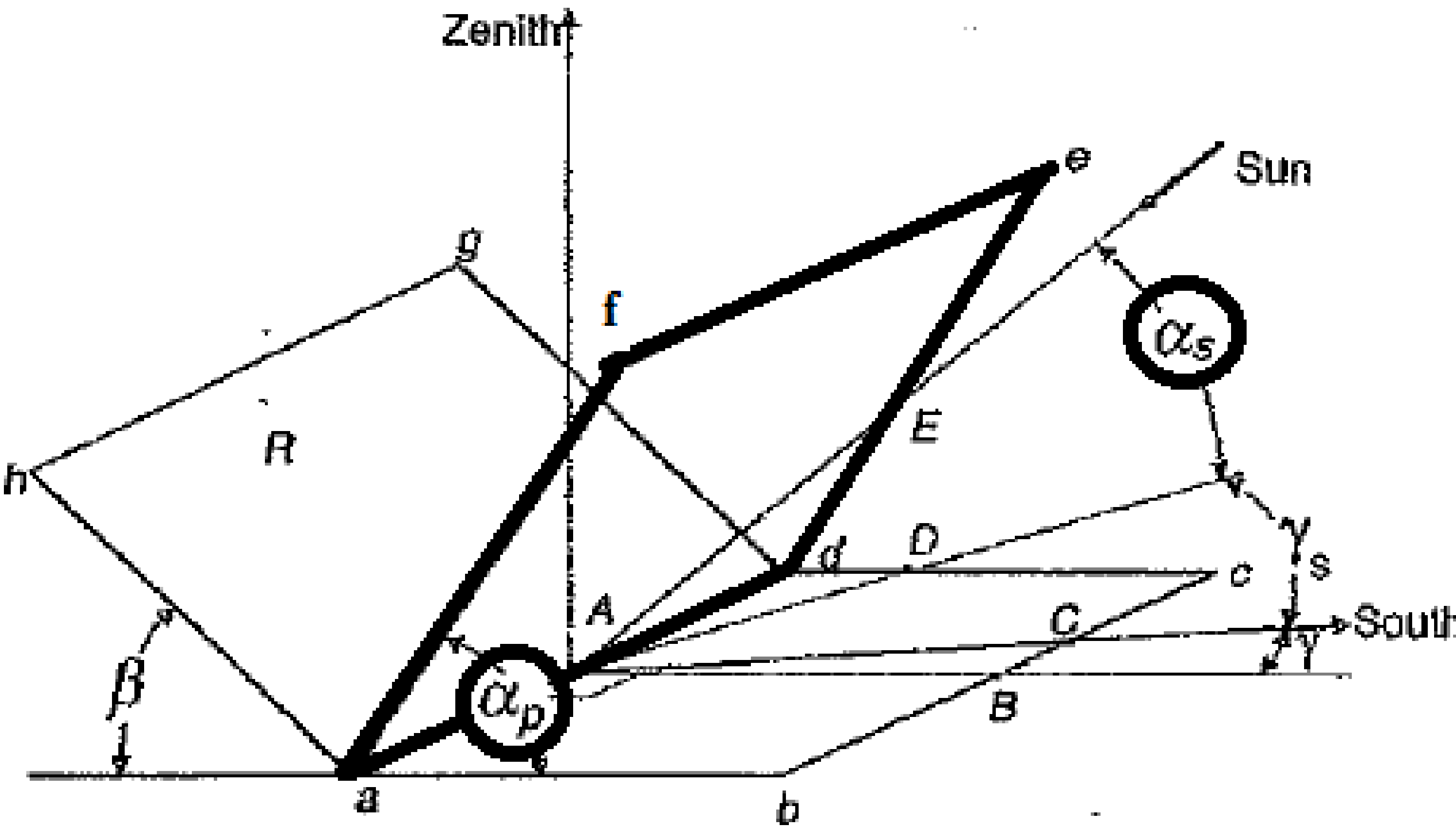
- Number of daylight hours (N) can be calculated as:

$$N = \frac{2}{15}\omega_s$$

For half-day (sunrise to noon or noon to sunrise), number of daylight hours will be half of above.

Profile angle

It is the angle through which a plane that is initially horizontal must be rotated about an axis in the plane of the given surface in order to include the sun.



Profile angle

- It is denoted by α_p and can be calculated as follow:

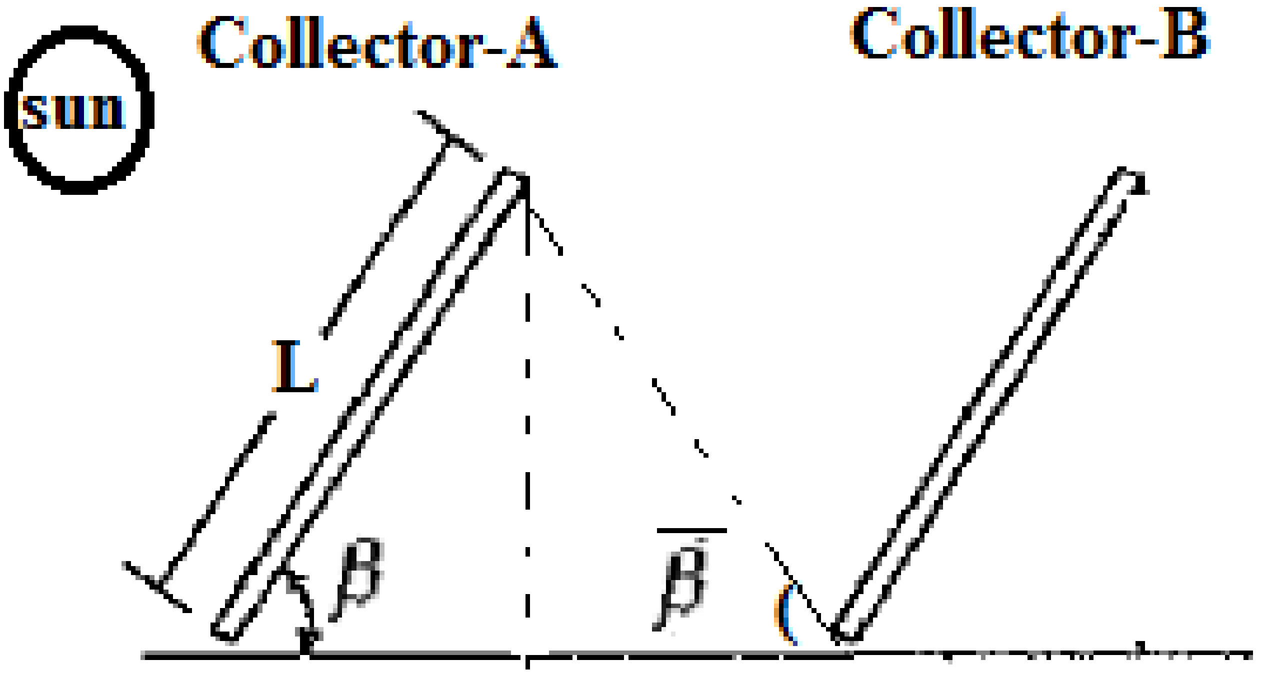
$$\tan \alpha_p = \frac{\tan \alpha_s}{\cos(\gamma_s - \gamma)}$$

- It is used in calculating shade of one collector (row) on to the next collector (row).
- In this way, profile angle can also be used in calculating the minimum distance between collector (rows).

Profile angle

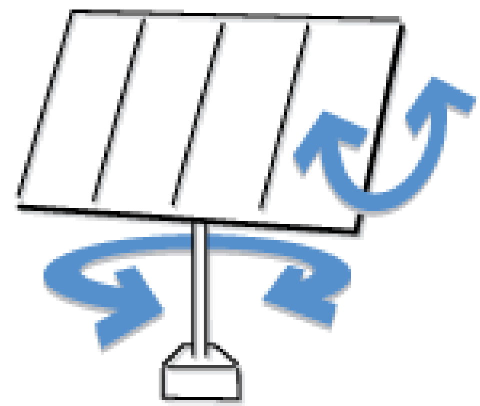
- Collector-B will be in shade of collector-A, only when:

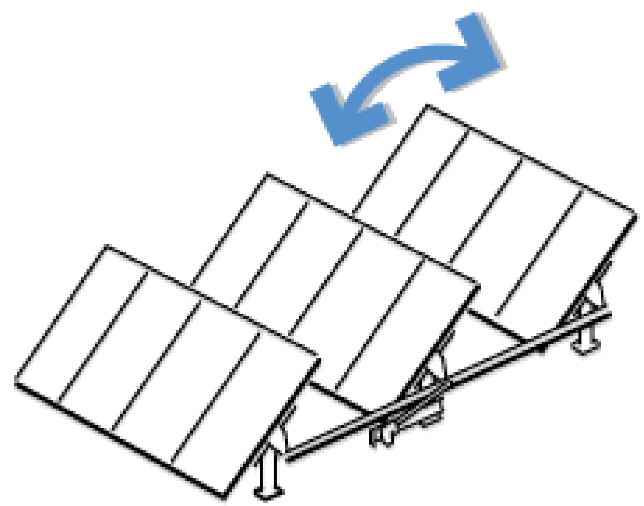
$$\alpha_p < \bar{\beta}$$



Angles for tracking surfaces

- Some solar collectors "track" the sun by moving in prescribed ways to minimize the angle of incidence of beam radiation on their surfaces and thus maximize the incident beam radiation.
- Tracking the sun is much more essential in concentrating systems e.g. parabolic troughs and dishes.





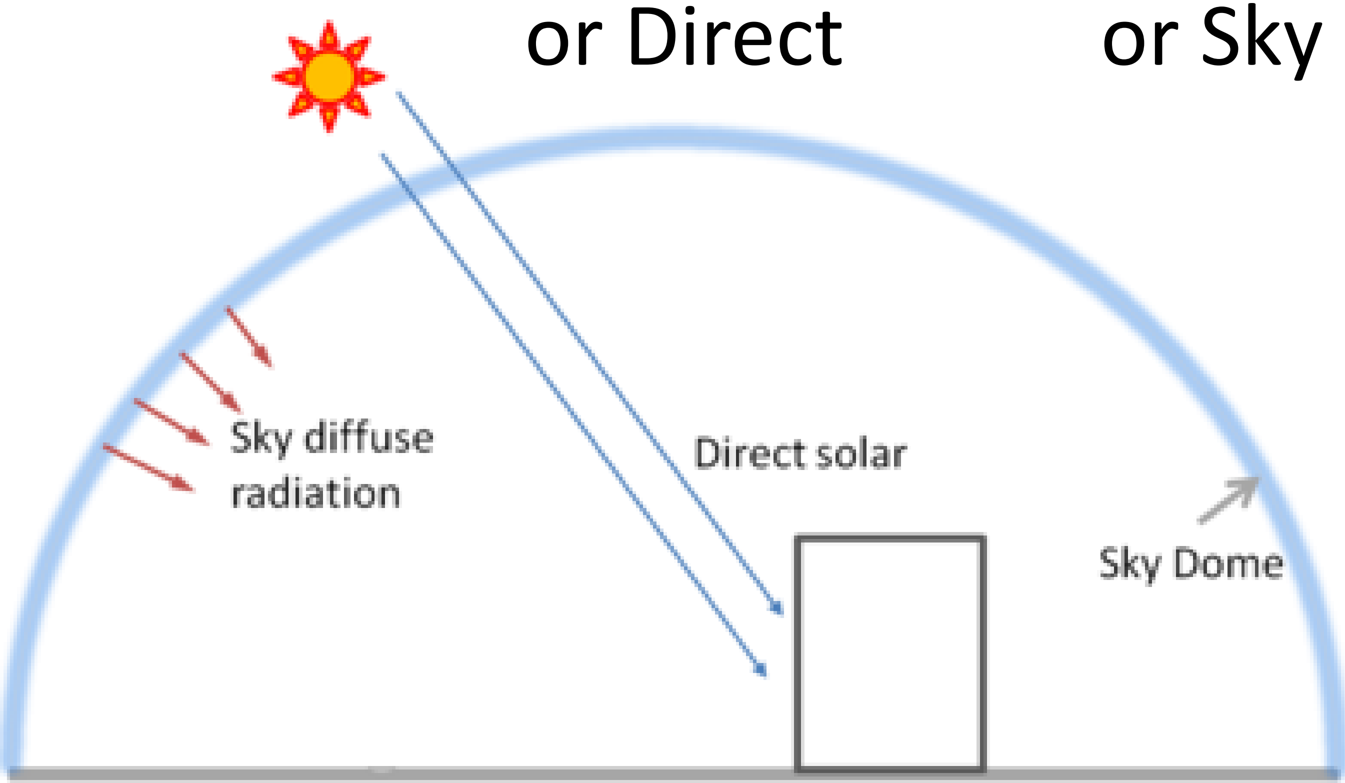
(See "Tracking surfaces" in *Reference Information*)

Types of solar radiations

1. Types by components:

Total = **Beam** + **Diffuse**

or Direct   or Sky



Types of solar radiations

2. Types by terrestre:

Extraterrestrial	**Terrestrial**
• Solar radiations received on earth without the presence of atmosphere OR solar radiations received outside earth atmosphere. • We always **calculate** these radiations.	• Solar radiations received on earth in the presence of atmosphere. • We can **measure** or **estimate** these radiations. Ready **databases** are also available e.g. TMY.

Measurement of solar radiations

1. Magnitude of solar radiations:

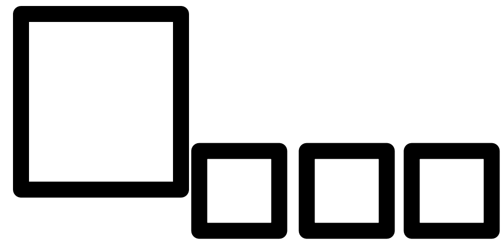
Irradiance	Irradiation/Insolation
• *Rate of energy* (power) received per unit area • Symbol: **G** • Unit: W/m^2	*Energy* received per unit area in a given time → Hourly: **I**, Unit: J/m^2 → Daily: **H**, Unit: J/m^2 → Monthly avg. daily: $\bar{\mathbf{H}}$, Unit: J/m^2

Measurement of solar radiations

2. Tilt (β) and orientation (γ) of measuring instrument:

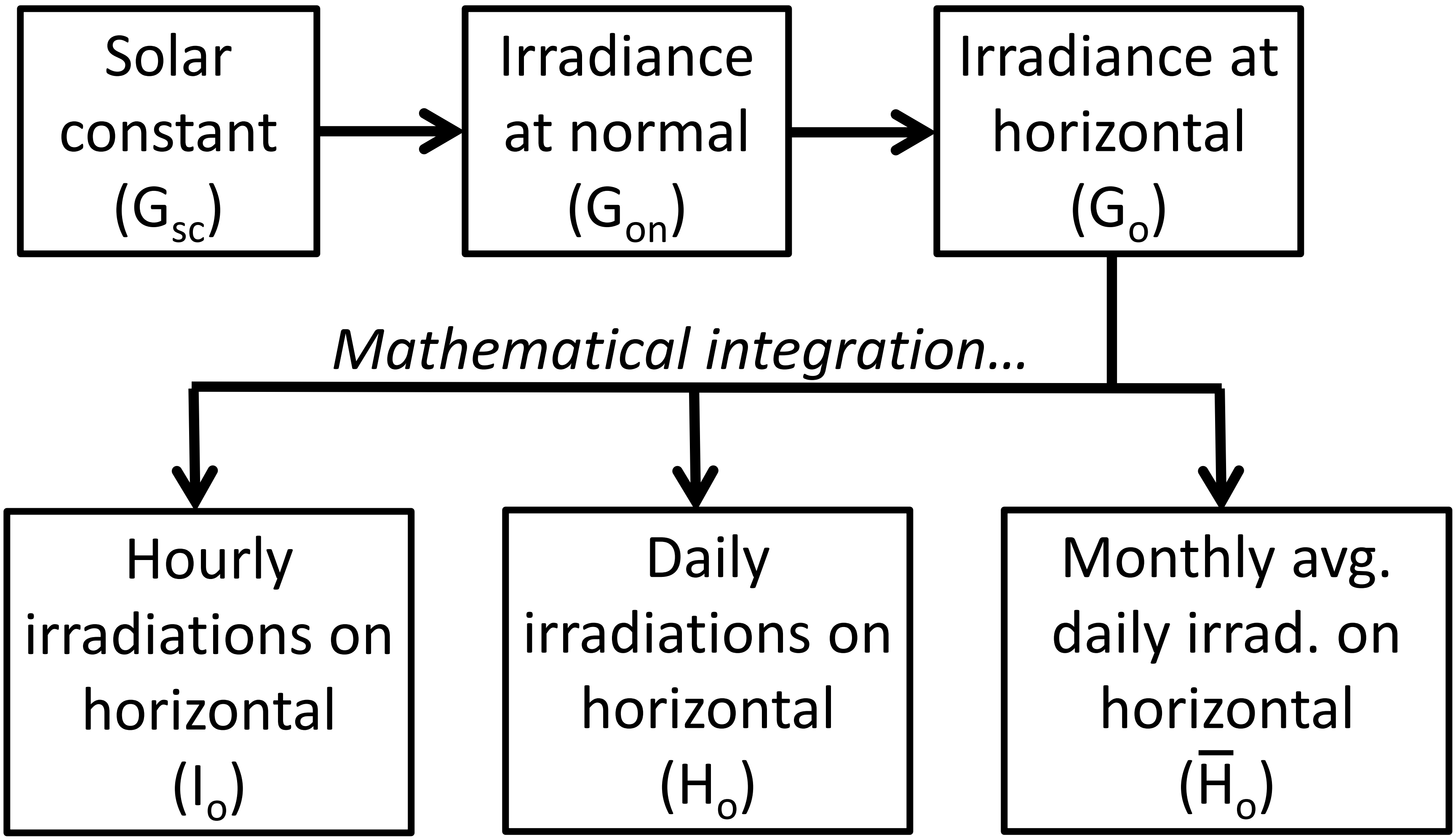
- Horizontal (β=0°, irrespective of γ)
- Normal to sun ($\beta=\theta_z$, $\gamma=\gamma_s$)
- Tilt (any β, γ is usually 0°)
- Latitude ($\beta=\phi$, γ is usually 0°)

Representation of solar radiations



- Symbols:
  - Irradiance: **G**
  - Irradiations:
    I (hourly), **H** (daily), $\bar{\mathbf{H}}$ (monthly average daily)
- Subscripts:
  - Ex.terr.: **o** Terrestrial: -
  - Beam: **b** Diffuse: **d** Total -
  - Normal: **n** Tilt: **T** Horizontal -

Extraterrestrial solar radiations



Solar constant (G_{sc})

Extraterrestrial solar radiations received at normal, when earth is at an average distance (1 au) away from sun.

$$G_{sc} = 1367\ W/m^2$$

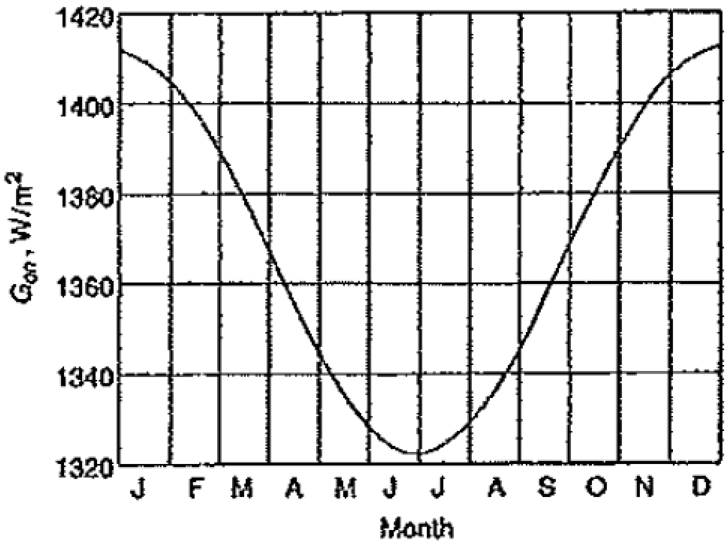
Adopted by World Radiation Center (WRC)

G_{sc}

Ex.terr. irradiance at normal

Extraterrestrial solar radiations received at normal. It deviates from G_{SC} as the earth move near or away from the sun.

$$G_{on} = G_{sc}\left(1 + 0.033\cos\frac{360n}{365}\right)$$



Ex.terr. irradiance on horizontal

Extraterrestrial solar radiations received at horizontal. It is derived from G_{on} and therefore, it deviates from G_{SC} as the earth move near or away from the sun.

$$G_o = G_{on} \times (\cos \emptyset \cos \delta \cos \omega + \sin \emptyset \sin \delta)$$

G_o

Ex.terr. hourly irradiation on horizontal

$$I_o = \frac{12 \times 3600}{\pi} G_{sc} \times \left(1 + 0.033 \cos \frac{360n}{365}\right) \times \left[\cos \emptyset \cos \delta \left(\sin \omega_2 - \sin \omega_1\right)\right.$$

I_o

Ex.terr. daily irradiation on horizontal

$$H_o = \frac{24 \times 3600}{\pi} G_{sc} \times \left(1 + 0.033 \cos \frac{360n}{365}\right) \times \left[\cos \emptyset \cos \delta \sin \omega_s + \frac{\pi \omega_s}{180} \sin \emptyset \sin \delta\right]$$

H_o

Ex.terr. monthly average daily irradiation on horizontal

$$\bar{H}_o = \frac{24 \times 3600}{\pi} G_{sc} \times \left(1 + 0.033 \cos \frac{360n}{365}\right) \times \left[\cos \emptyset \cos \delta \sin \omega_s + \frac{\pi \omega_s}{180} \sin \emptyset \sin \delta\right]$$

Where day and time dependent parameters are calculated on average day of a particular month i.e. $n = \bar{n}$

$\bar{\mathbf{H}}_{\mathbf{o}}$

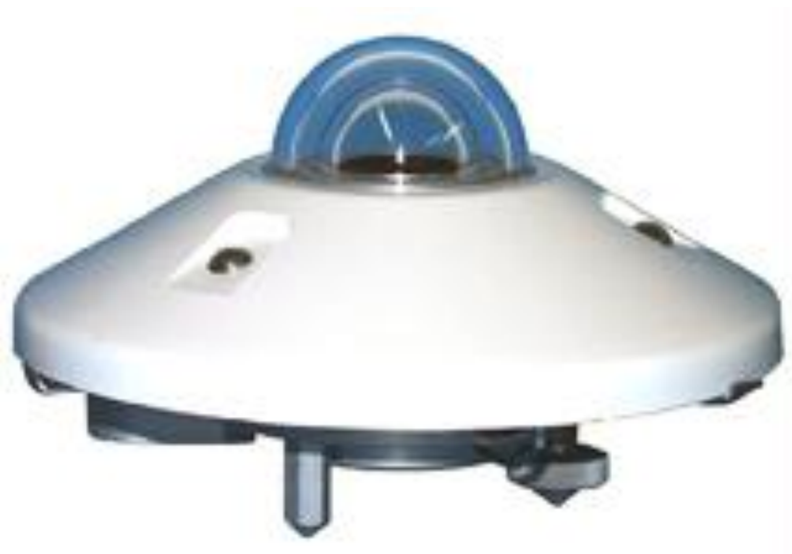
Terrestrial radiations

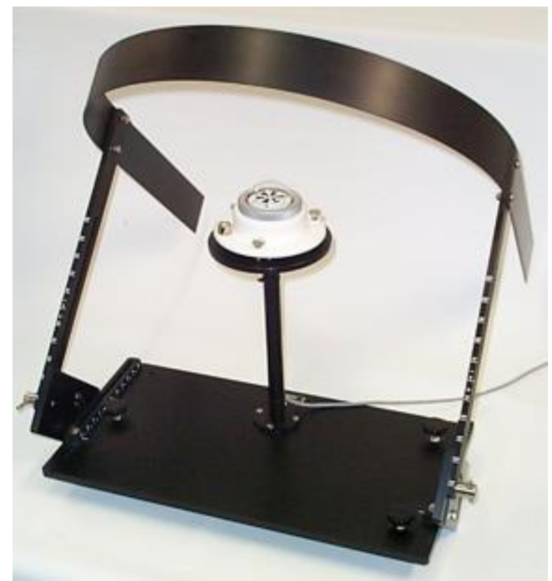
Can be…

- measured by instruments
- obtained from databases e.g. TMY, NASA SSE etc.
- estimated by different correlations

Terrestrial radiations measurement

- Total irradiance can be measured using **Pyranometer**
- Diffuse irradiance can be measured using **Pyranometer with shading ring**





Terrestrial radiations measurement

- Beam irradiance can be measured using **Pyrheliometer**

- Beam irradiance can also be measured by taking difference in readings of pyranometer with and without shadow band:

beam = total - diffuse

Terrestrial radiations databases

1. NASA SSE:

Monthly average daily total irradiation on horizontal surface ($\overline{H}$) can be obtained from NASA Surface meteorology and Solar Energy (SSE) Database, accessible from:

http://eosweb.larc.nasa.gov/sse/RETScreen/

(See "NASA SSE" in Reference Information)

Terrestrial radiations databases

2. TMY files:

Information about hourly solar radiations can be obtained from Typical Meteorological Year files.

(See "TMY" section in Reference Information)

Terrestrial irradiation estimation

- *Angstrom-type* regression equations are generally used:

$$\frac{\bar{H}}{\bar{\bar{H}}_o} = a + b\,\frac{\bar{n}}{\bar{\bar{N}}}$$

(See "Terrestrial Radiations Estimations" section in Reference Information)

Terrestrial irradiation estimation

For Karachi:

$$\frac{\bar{H}}{\bar{H}_o} = 0.324 + 0.405\frac{\bar{n}}{\bar{N}}$$

Where,

$\bar{n}$ is the representation of cloud cover and $\bar{N}$ is the day length of average day of month.

Month	$\overline{n}/\overline{N}$
Jan	0.805
Feb	0.776
Mar	0.762
Apr	0.738
May	0.743
Jun	0.595
Jul	0.381
Aug	0.390
Sep	0.602
Oct	0.818
Nov	0.837
Dec	0.830

Clearness index

- A ratio which mathematically represents sky clearness.

  =1 (clear day)

  <1 (not clear day)

- Used for finding:
  - frequency distribution of various radiation levels
  - diffuse components from total irradiations

Clearness index

1. Hourly clearness index:

$$k_T = \frac{I}{I_o}$$

2. Daily clearness index:

$$K_T = \frac{H}{H_o}$$

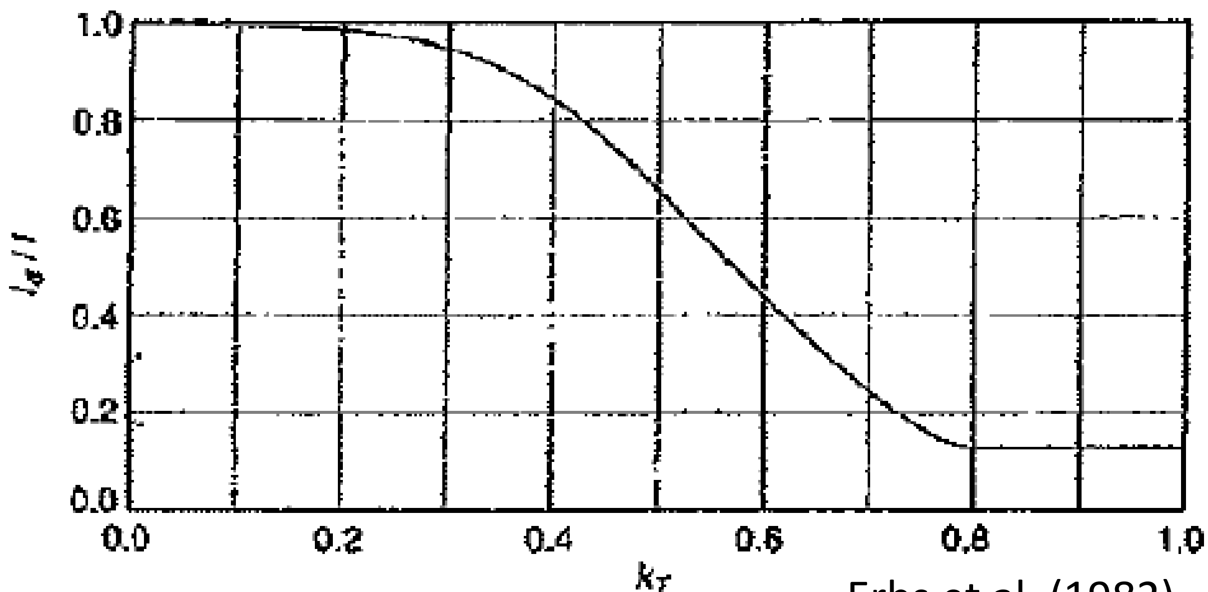
3. Monthly average daily clearness index:

$$\overline{K}_T = \frac{\overline{H}}{\overline{H}_o}$$

Diffuse component of hourly irradiation (on horizontal)

Orgill and Holland correlation:

$$\frac{I_d}{I} = \begin{cases} 1 - 0.249k_T, & k_T \leq 0.35 \\ 1.557 - 1.84k_T, & 0.35 < k_T < 0.75 \\ 0.177, & k_T \geq 0.75 \end{cases}$$



Erbs et al. (1982)

Diffuse component of daily irradiation (on horizontal)

Collares-Pereira and Rabl correlation:

$$\frac{H_d}{H} = \begin{cases} 0.99, & K_T \leq 0.17 \\ \left.\begin{matrix} 1.188 - 2.272K_T \\ +9.473{K_T}^2 \\ -21.865{K_T}^3 \\ +14.648{K_T}^4 \end{matrix}\right\}, & 0.17 < K_T < 0.75 \\ -0.54K_T + 0.632, & 0.75 < K_T < 0.8 \\ 0.2, & K_T \geq 0.8 \end{cases}$$

Diffuse component of monthly average daily irradiation (on horizontal)

Collares-Pereira and Rabl correlation:

$$\frac{\overline{H}_d}{\overline{\overline{H}}} = 0.775 + 0.00606(\omega_s - 90) - [0.505$$

Hourly total irradiation from daily irradiation (on horizontal)

For any mid-point (ω) of an hour,

$$I = r_t H$$

According to Collares-Pereira and Rabl:

$$r_t = \frac{\pi}{24}(a + b\cos\omega)\frac{\cos\omega - \cos\omega_s}{\sin\omega_s - \frac{\pi\omega_s}{180}\cos\omega_s}$$

Where,

$$a = 0.409 + 0.5016\sin(\omega_s - 60)$$

$$b = 0.6609 - 0.4767\sin(\omega_s - 60)$$

Hourly diffuse irradiations from daily diffuse irradiation (on horizontal)

For any mid-point (ω) of an hour,

$$I_d = r_d H_d$$

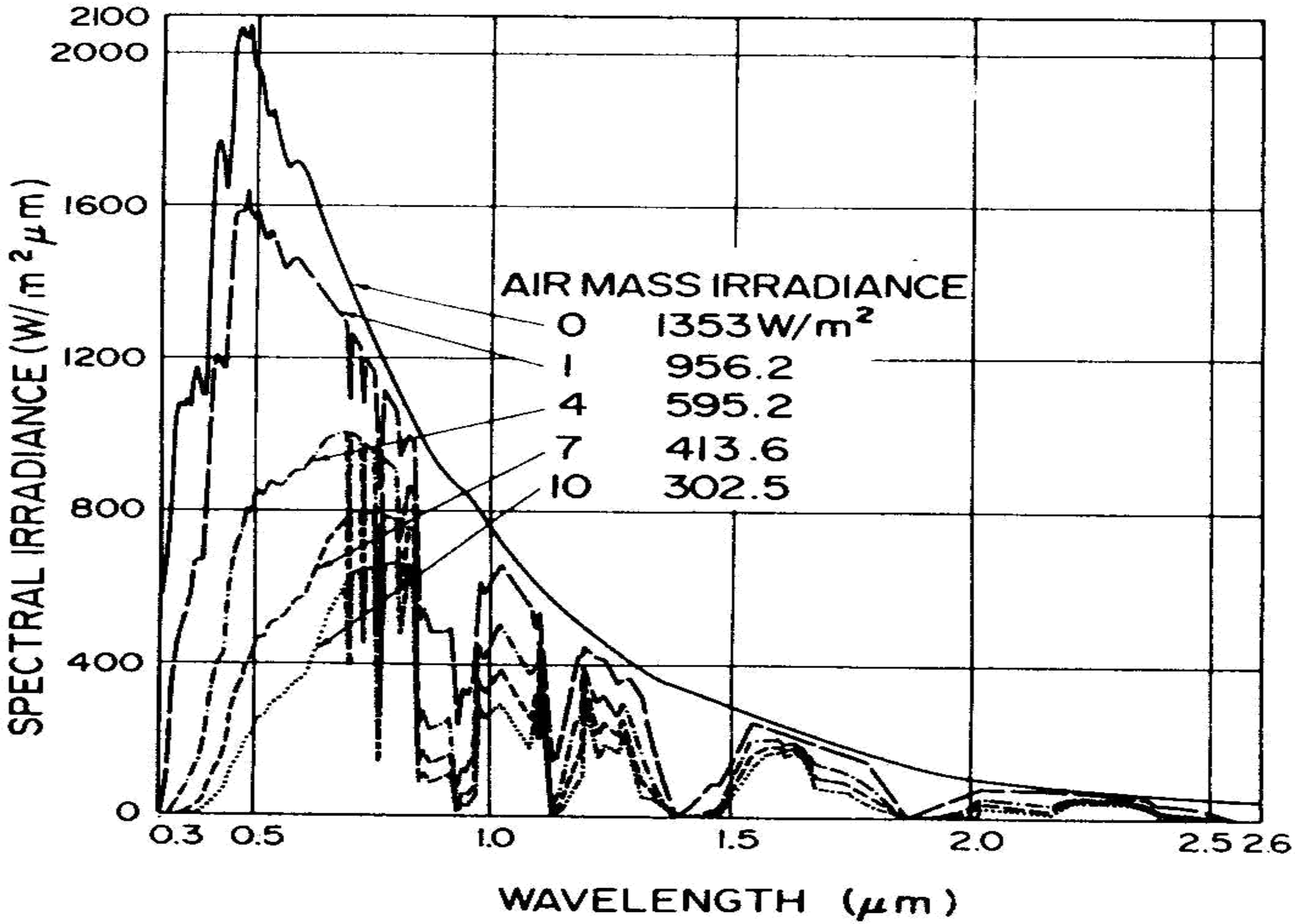
From Liu and Jordan:

$$r_d = \frac{\pi}{24} \frac{\cos\omega - \cos\omega_s}{\sin\omega_s - \frac{\pi\omega_s}{180}\cos\omega_s}$$

Air mass and radiations

- Terrestrial radiations depends upon the path length travelled through atmosphere. Hence, these radiations can be characterized by air mass (AM).
- Extraterrestrial solar radiations are symbolized as **AM0**.
- For different air masses, spectral distribution of solar radiations is different.

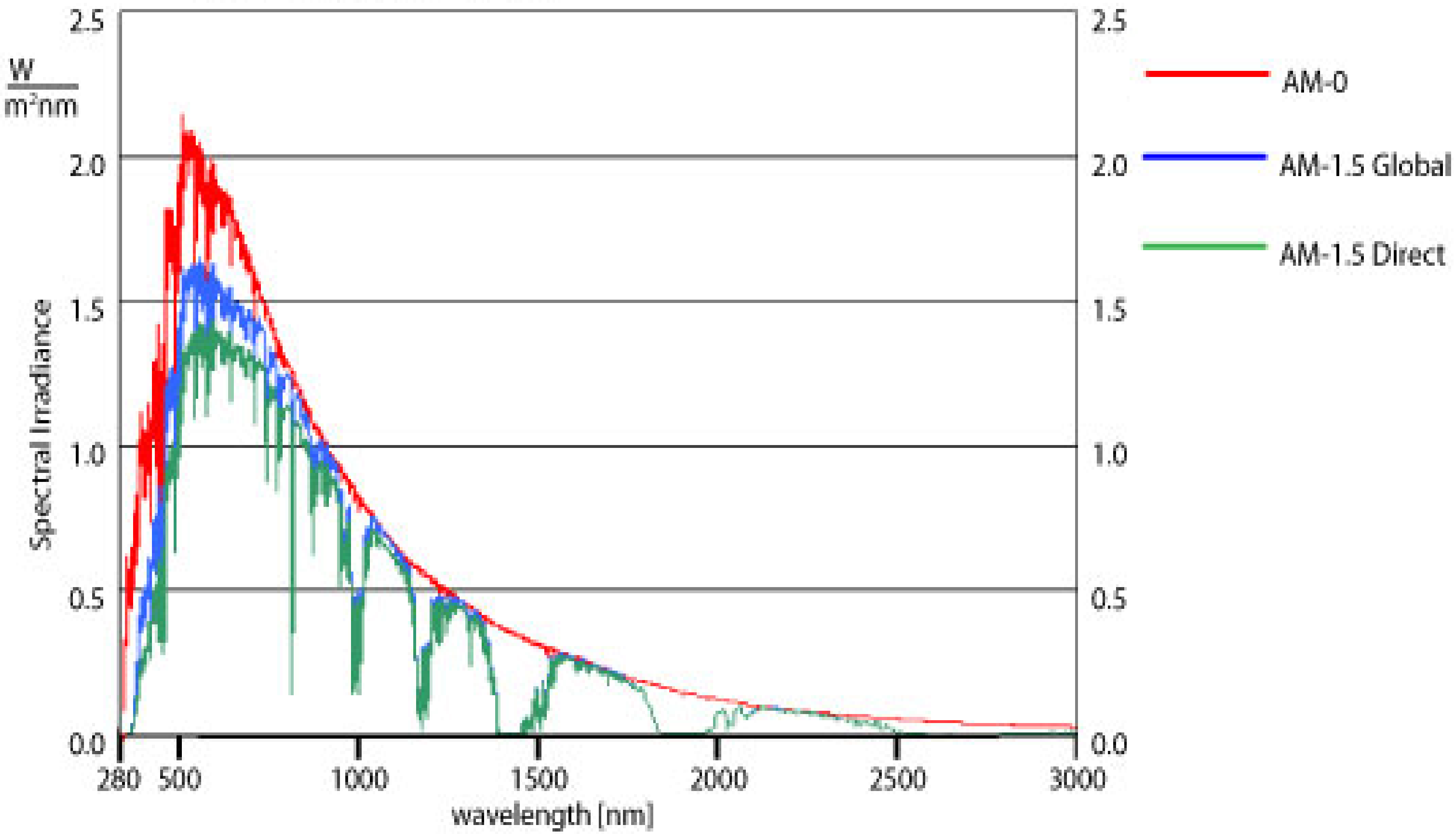
Air mass and radiations



Air mass and radiations

- The standard spectrum at the Earth's surface generally used are:
  - **AM1.5G**, (G = global)
  - **AM1.5D** (D = direct radiation only)
- AM1.5D = 28% of AM0
  18% (absorption) + 10% (scattering).
- AM1.5G = 110% AM1.5D = 970 W/m^2.

Air mass and radiations

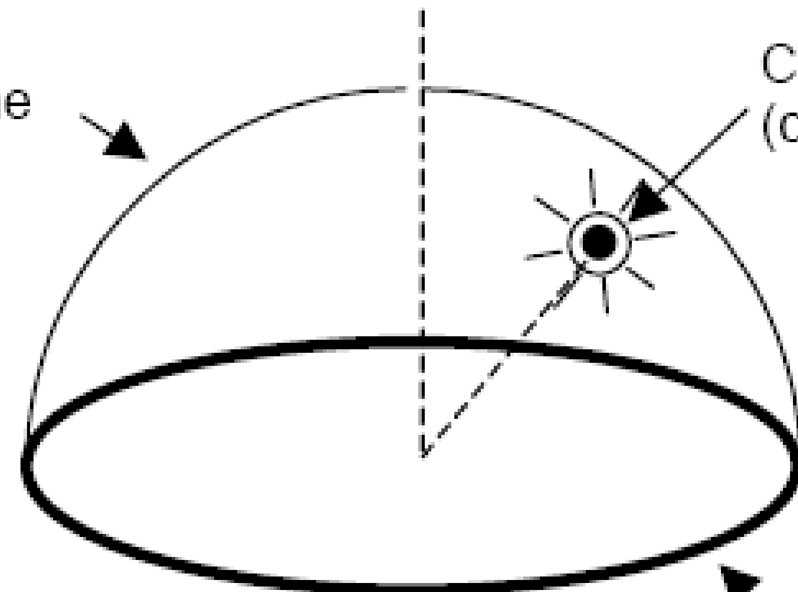


Radiations on a tilted plane

To calculate radiations on a tilted plane, following information are required:

- tilt angle
- total, beam and diffused components of radiations on horizontal (at least two of these)
- diffuse sky assumptions (isotropic or anisotropic)
- calculation model

Diffuse sky assumptions



Diffuse sky assumptions

Diffuse radiations consist of three parts:

1. Isotropic *(represented by: iso)*
2. Circumsolar brightening *(represented by : cs)*
3. Horizon brightening *(represented by : hz)*

There are two types of diffuse sky assumptions:

1. Isotropic sky (iso)
2. Anisotropic sky (iso + cs, iso + cs + hz)

General calculation model

$$X_T = X_b R_b + X_{d,iso} F_{c-s} + X_{d,cs} R_b + X_{d,hz} F_{c-hz} + X \rho_g F_{c-g}$$

Where,

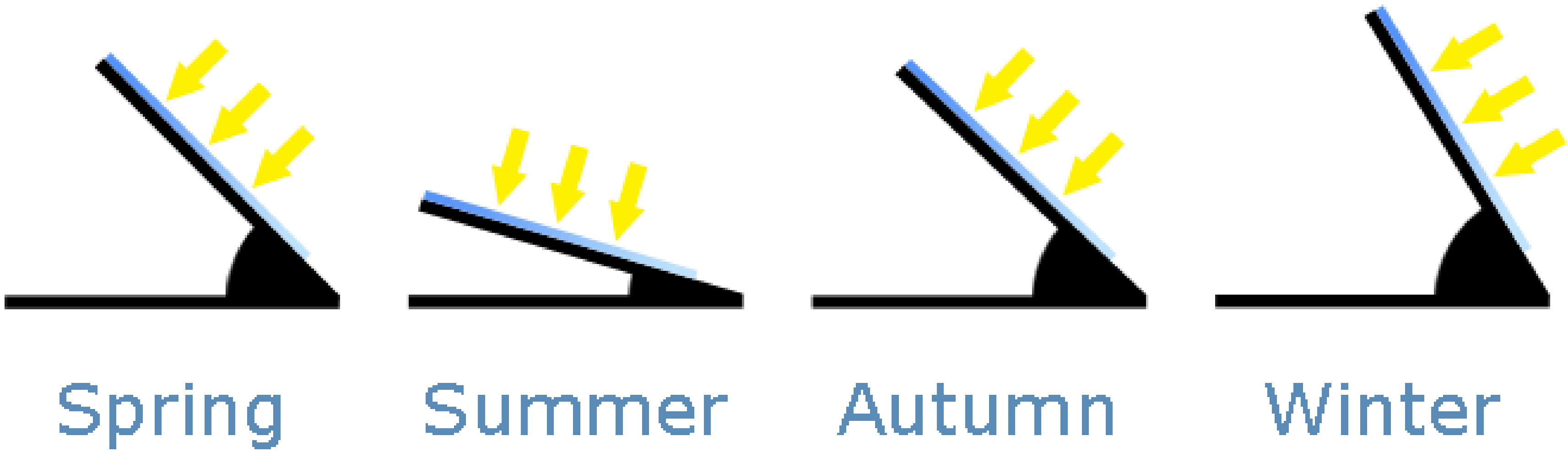
- X, X_b, X_d: total, beam and diffuse radiations (irradiance or irradiation) on horizontal
- iso, cs and hz: isotropic, circumsolar and horizon brightening parts of diffuse radiations
- R_b: beam radiations on tilt to horizontal ratio
- F_{c-s}, F_{c-hz} and F_{c-g}: shape factors from collector to sky, horizon and ground respectively
- ρ_g: albedo

Calculation models

1. Liu and Jordan (LJ) model (iso, γ=0°, I)
2. Liu and Jordan (LJ) model (iso, γ=0°, $\overline{H}$)
3. Hay and Davies (HD) model (iso+cs, γ=0°, I)
4. Hay, Davies, Klucher and Reindl (HDKR) model (iso+cs+hz, γ=0°, I)
5. Perez model (iso+cs+hz, γ=0°,I)
6. Klein and Theilacker (K-T) model (iso+cs, γ=0°, $\overline{H}$)
7. Klein and Theilacker (K-T) model (iso+cs, $\overline{H}$)

(See "Sky models" in Reference Information)

Optimum tilt angle

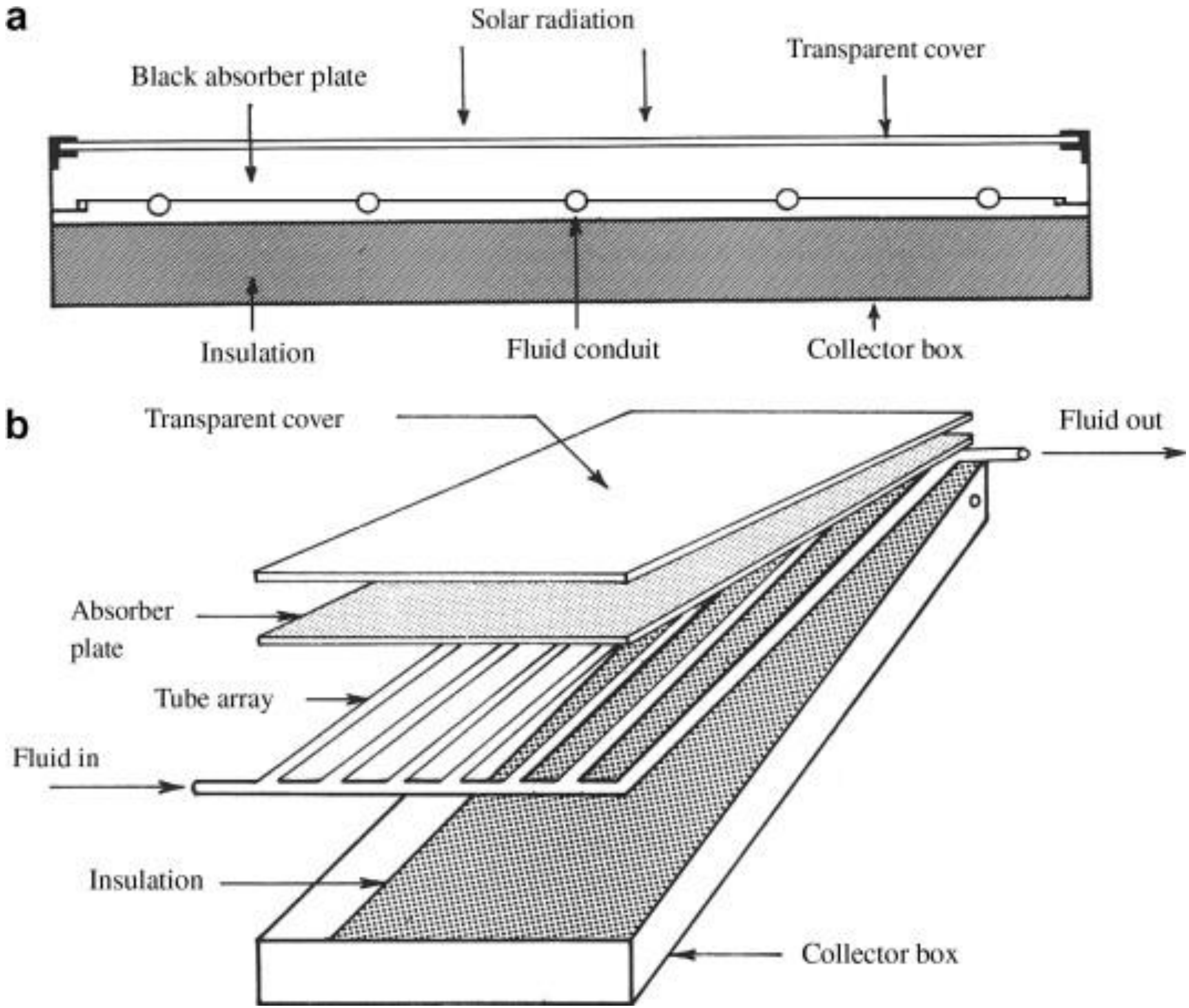


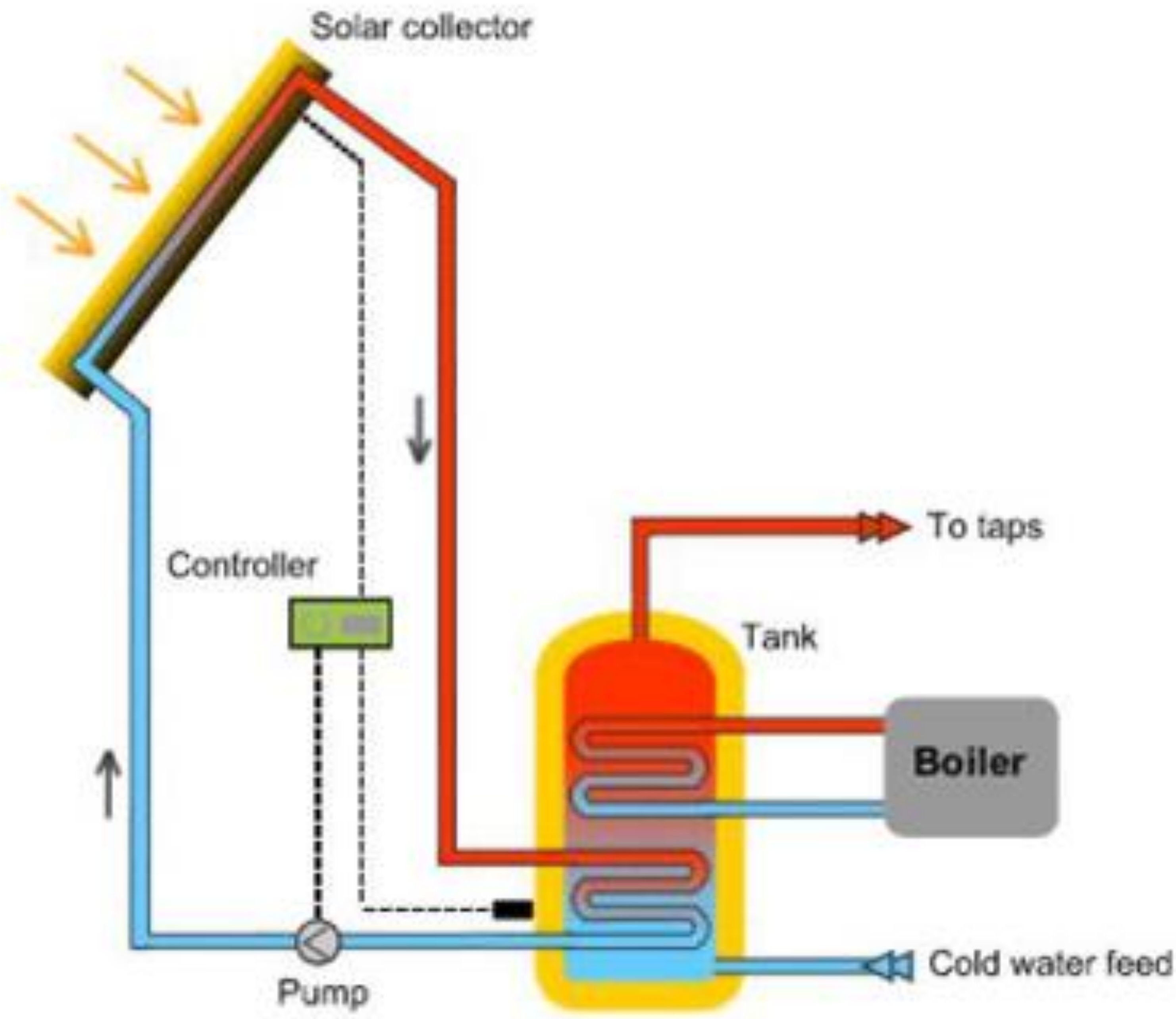
Introduction

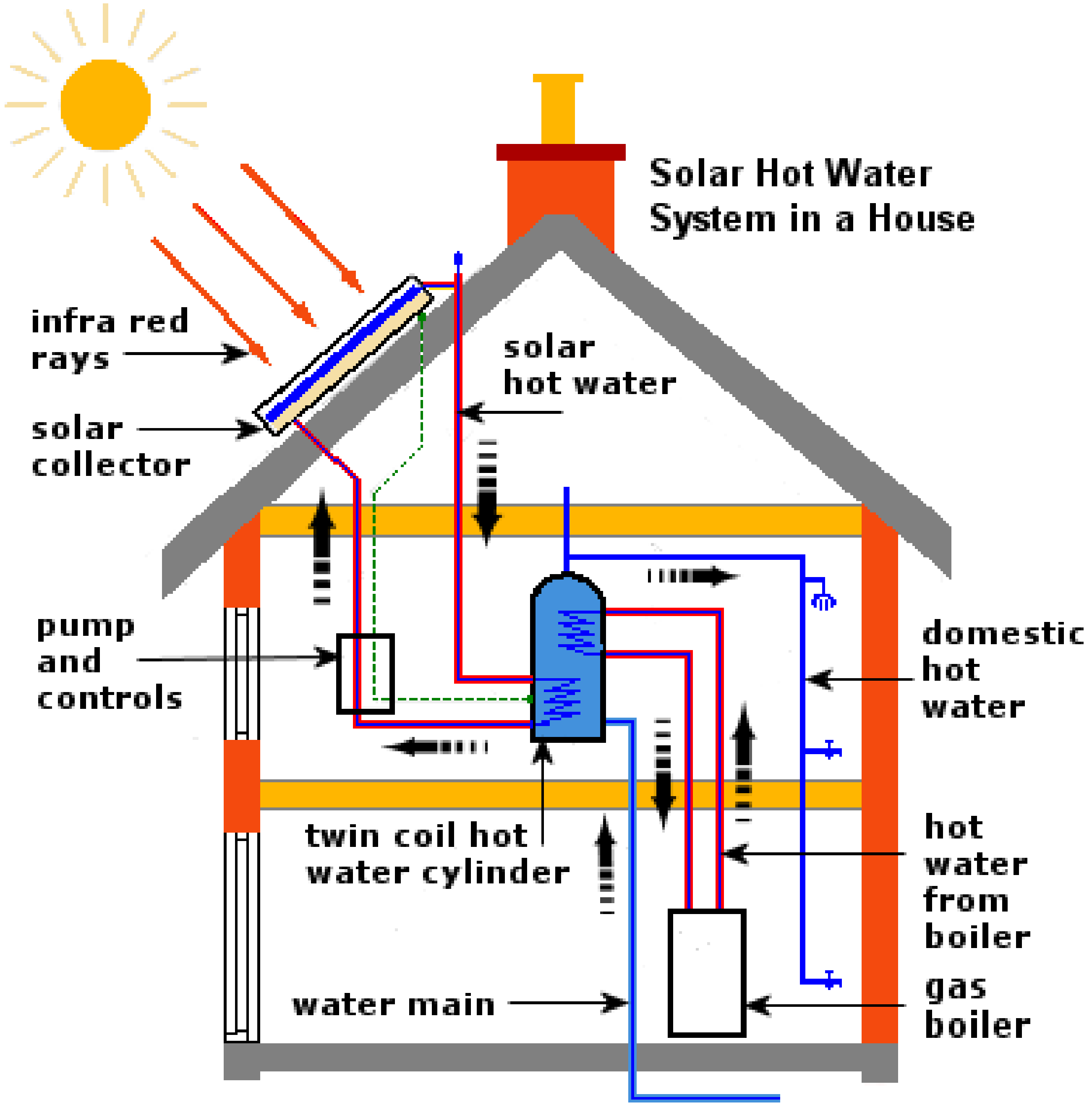
1. Flat-plate collectors are special type of heat-exchangers
2. Energy is transferred to fluid from a distant source of radiant energy
3. Incident solar radiations is not more than 1100 W/m^2 and is also variable
4. Designed for applications requiring energy delivery up to 100°C above ambient temperature.

Introduction

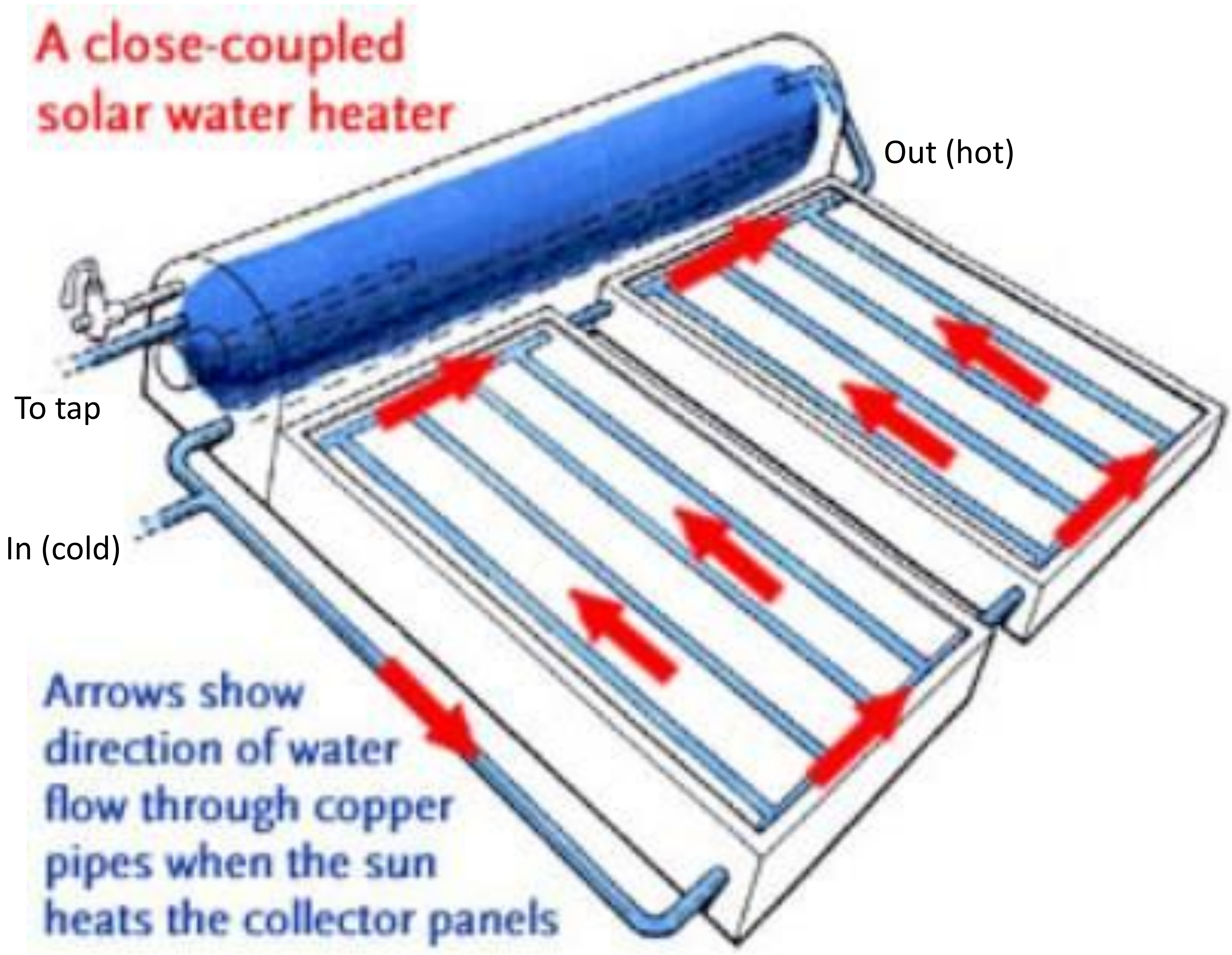
1. Use both beam and diffuse solar radiation
2. Do not require sun tracking and thus require low maintenance
3. Major applications: solar water heating, building heating, air conditioning and industrial process heat.


a
Solar radiation
Transparent cover
Black absorber plate
Insulation
Fluid conduit
Collector box
b
Transparent cover
Fluid out
Absorber plate
Tube array
Fluid in
Insulation
Collector box


Solar collector
Controller
To taps
Tank
Boiler
Pump
Cold water feed



Solar Hot Water
System in a House
infra red
rays
solar
collector
solar
hot water
pump
and
controls
domestic
hot
water
twin coil hot
water cylinder
hot
water
from
boiler
water main
gas
boiler


A close-coupled
solar water heater
Out (hot)
To tap
In (cold)
Arrows show
direction of water
flow through copper
pipes when the sun
heats the collector panels

Installation of flat-plate collectors at Mechanical Engineering Department, NED University of Engg. & Tech., Pakistan

Heat transfer: Fundamental

Heat transfer, in general:

$q = Q/A = (T_1 - T_2)/R = \Delta T/R = U\Delta T$ [W/m^2]

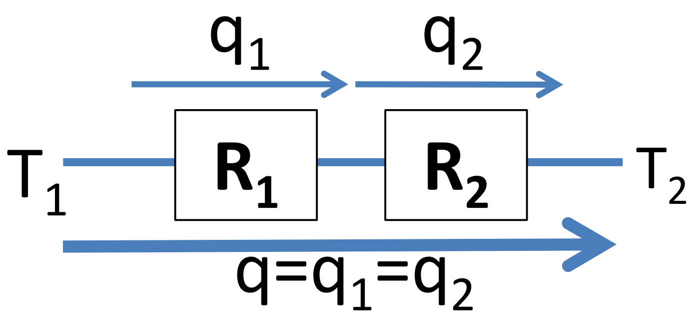
Where,

$T_1 > T_2$: Heat is transferred from higher to lower temperature

ΔT is the temperature difference	[K]
R is the thermal resistance	[m^{2}K/W]
A is the heat transfer area	[m^2]
U is overall H.T. coeff. $U=1/R$	[W/m^{2}K]

Heat transfer: Circuits

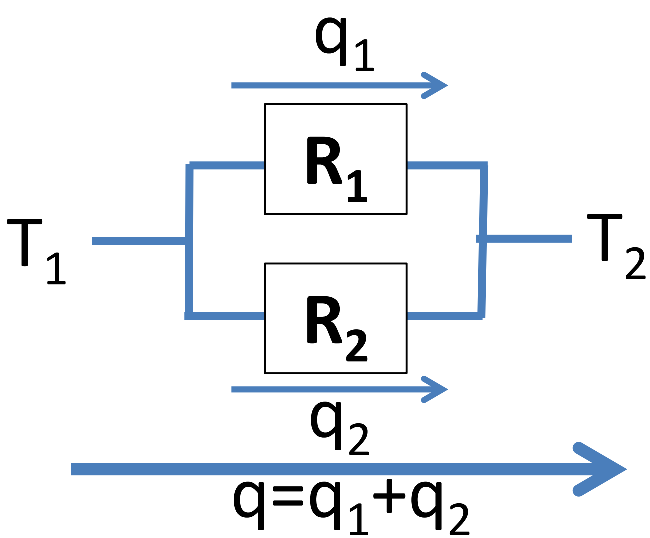
Resistances in series:



$$R = R_1 + R_2$$

$$U = \frac{1}{R_1 + R_2}$$

Resistances in parallel:

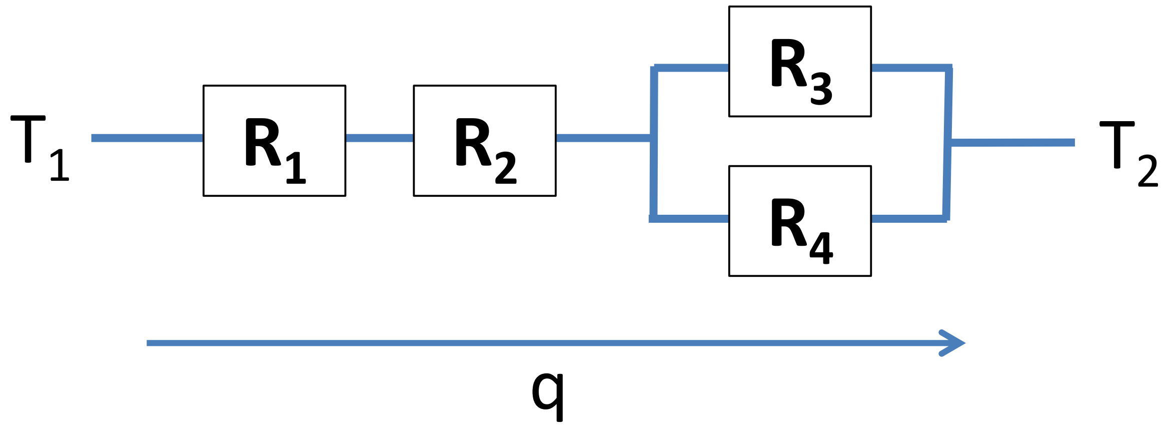


$$R = \frac{1}{{}^{1}/_{R_1} + {}^{1}/_{R_2}}$$

$$U = {}^{1}/_{R_1} + {}^{1}/_{R_2}$$

Example-1
Heat transfer: Circuits

Determine the heat transfer per unit area(q) and overall heat transfer coefficient (U) for the following circuit:



Heat transfer: Radiation

Radiation heat transfer between two infinite parallel plates:

$$\boldsymbol{R_r = 1/h_r}$$

and,

$$\boldsymbol{h_r = \frac{\sigma\left({T_1}^2 + {T_2}^2\right)(T_1 + T_2)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}}$$

Where,

$\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4

ϵ is the emissivity of a plate

Heat transfer: Radiation

Radiation heat transfer between a small object surrounded by a large enclosure:

$$\boldsymbol{R_r = 1/h_r}$$

and,

$$\boldsymbol{h_r = \frac{\sigma\left({T_1}^2 + {T_2}^2\right)(T_1 + T_2)}{1/\varepsilon}}$$

$$\boldsymbol{= \varepsilon\sigma\left({T_1}^2 + {T_2}^2\right)(T_1 + T_2)} \qquad \text{[W/m}^2\text{K]}$$

Heat transfer: Sky Temperature

1. Sky temperature is denoted by $\mathbf{T_s}$
2. Generally, $\mathbf{T_s = T_a}$ may be assumed because sky temperature does not make much difference in evaluating collector performance.
3. For a bit more accuracy:

   In hot climates: $\mathbf{T_s = T_a + 5°C}$

   In cold climates: $\mathbf{T_s = T_a + 10°C}$

Heat transfer: Convection

Convection heat transfer between parallel plates:

$$\boldsymbol{R_c = 1/h_c} \quad \text{and} \quad \boldsymbol{h_c = N_u k/L}$$

Where,

$$N_u = 1 + 1.44\left[1 - \frac{1708(\sin 1.8\beta)^{1.6}}{R_a \cos\beta}\right]\left[1 - \frac{1708}{R_a \cos\beta}\right]^{+} + \left[\left(\frac{R_a \cos\beta}{5830}\right)^{1/3} - 1\right]^{+}$$

Note: Above is valid for tilt angles between 0° to 75°.
'+' indicates that only positive values are to be considered. Negative values should be discarded.

Heat transfer: Convection

$$R_a = \frac{g\beta' \Delta T L^3}{\vartheta\alpha} \text{also} \;\; P_r = \vartheta/\alpha$$

Where,

Fluid properties are evaluated at mean temperature

Ra	Rayleigh number
Pr	Prandtl number
L	plate spacing
k	thermal conductivity
g	gravitational constant
β‘	volumetric coefficient of expansion

for ideal gas, β‘ = 1/T [K^{-1}]

ϑ, α kinematic viscosity and thermal diffusivity

Heat transfer: Conduction

Conduction heat transfer through a material:

$$\boldsymbol{R_k = L/k}$$

Where,

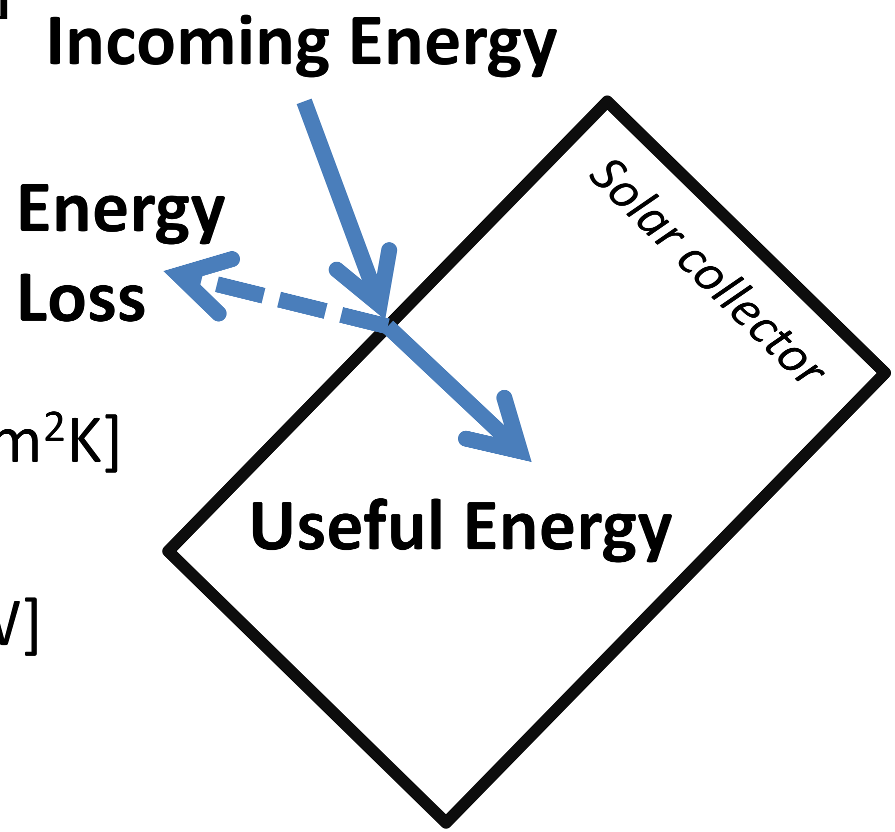
L	material thickness	[m]
k	thermal conductivity	[W/mK]

General energy balance equation

In steady-state:

Useful Energy = Incoming Energy – Energy Loss [W]

$$\boldsymbol{Q_u = A_c\left[S - U_L\left(T_{pm} - T_a\right)\right]}$$



A_c = Collector area [m^2]

T_{pm} = Absorber plate temp. [K]

T_a = Ambient temp. [K]

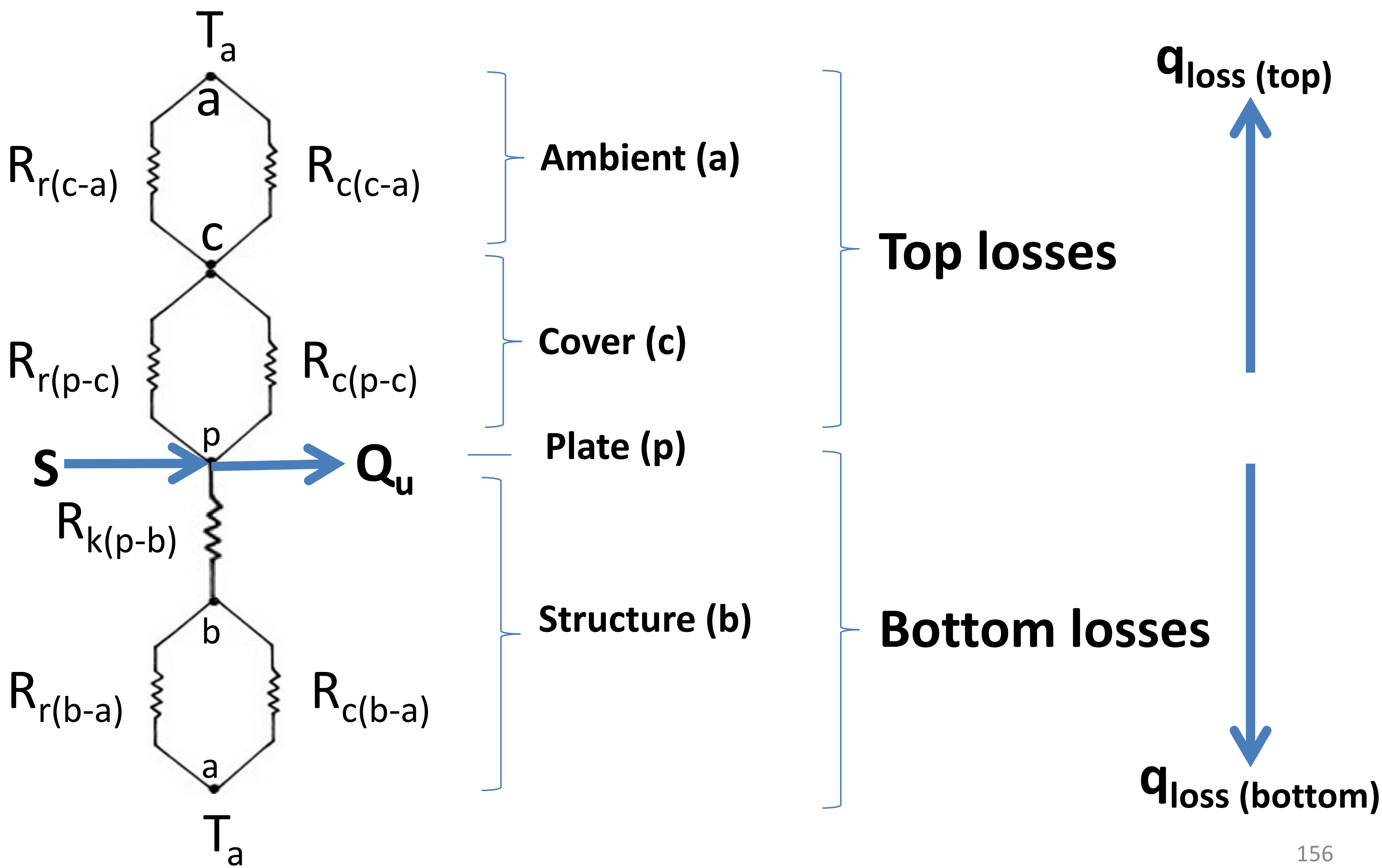
U_L = Overall heat loss coeff. [W/m^2K]

Q_u = Useful Energy [W]

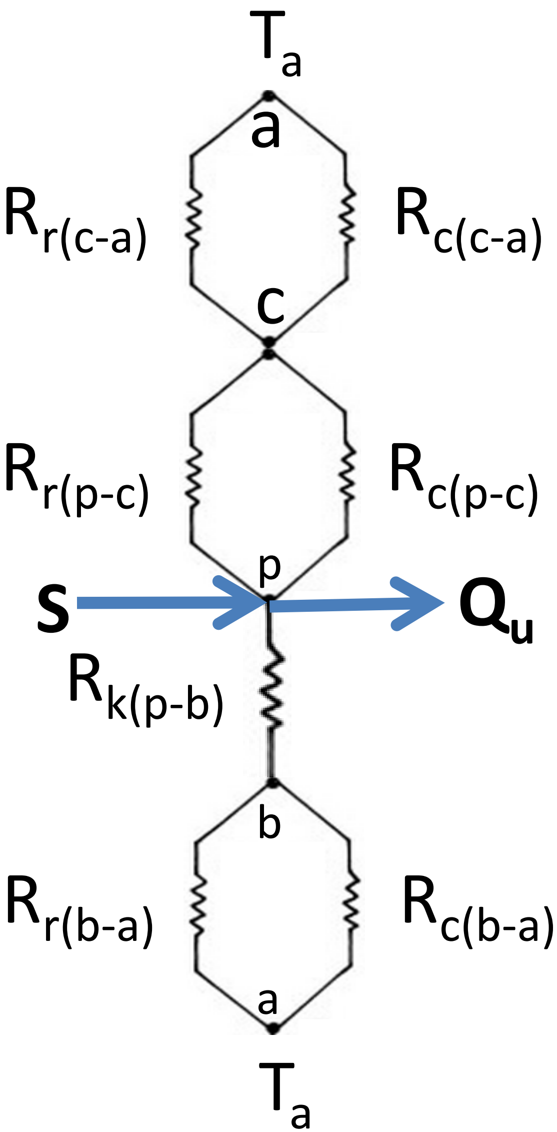
SA_c = Incoming (Solar) Energy [W]

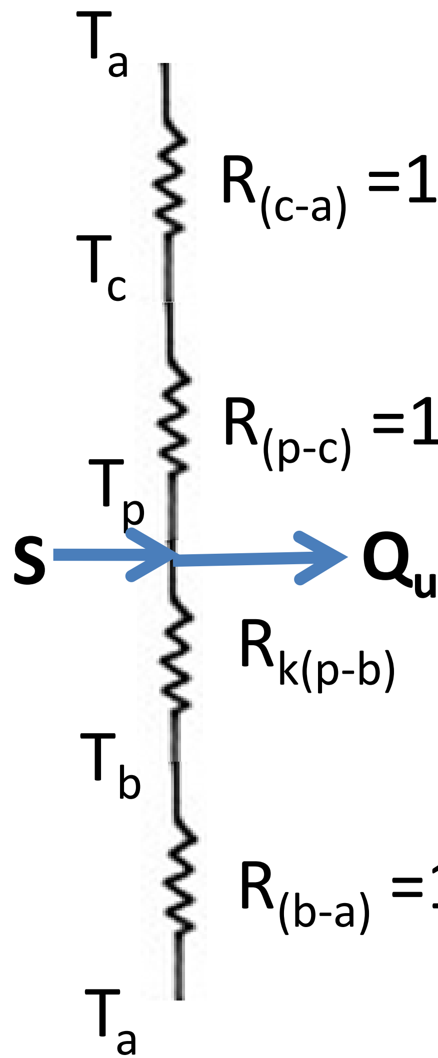
$A_cU_L(T_{pm}-T_a)$ = Energy Loss [W]

Thermal network diagram



Thermal network diagram



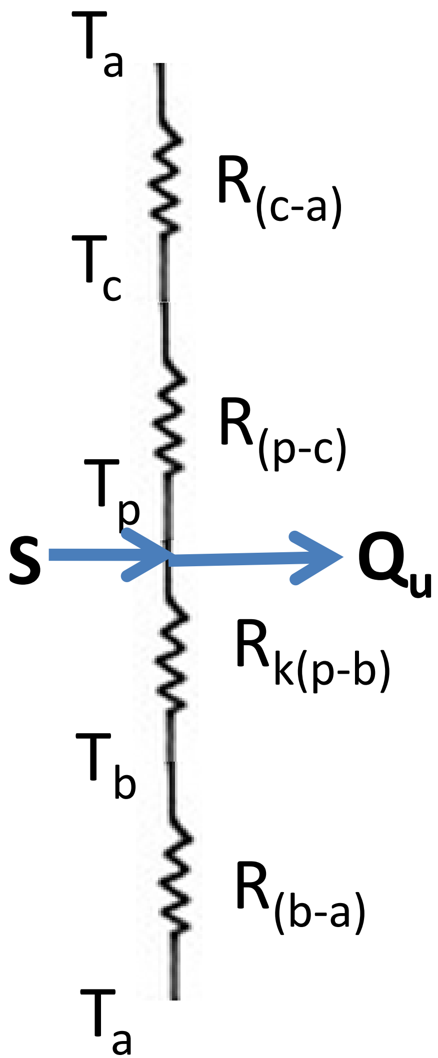


$$R_{(c\text{-}a)} = 1/(1/R_{r(c\text{-}a)} + 1/R_{c(c\text{-}a)})$$

$$R_{(p\text{-}c)} = 1/(1/R_{r(p\text{-}c)} + 1/R_{c(p\text{-}c)})$$

$$R_{(b\text{-}a)} = 1/(1/R_{r(b\text{-}a)} + 1/R_{c(b\text{-}a)})$$

Cover temperature



1. Ambient and plate temperatures are generally known.

2. U_{top} can be calculated as:

$$\mathbf{U_{top} = 1/(R_{(c-a)} + R_{(p-c)})}$$

3. From energy balance:

$$q_{p-c} = q_{p-a}$$

$$(T_p - T_c)/R_{(p-c)} = U_{top}(T_p - T_a)$$

$$\Rightarrow \mathbf{T_c = T_p - U_{top}\,(T_p - T_a) \times R_{(p-c)}}$$

Thermal resistances

$R_{r(c-a)} = 1/h_{r(c-a)} = 1/\varepsilon_c\sigma(T_a^2+T_c^2)\,(T_a+T_c)$

$R_{c(c-a)} = 1/h_{c(c-a)} = 1/h_w$

$R_{r(p-c)} = 1/h_{r(p-c)} = 1/[\sigma(T_c^2+T_p^2)\,(T_c+T_p)/(1/\varepsilon_c+ 1/\varepsilon_p-1)]$

$R_{c(p-c)} = 1/h_{c(p-c)} = 1/h_c$

$R_{k(p-b)} = L/k$

$R_{r(b-a)} = 1/h_{r(b-a)} = 1/\varepsilon_b\sigma(T_a^2+T_b^2)\,(T_a+T_b)$

$R_{c(b-a)} = 1/h_{c(b-a)} = 1/h_w$

Solution methodology

$$U_{top} = 1/(R_{(c-a)} + R_{(p-c)})$$

Cover to ambient

$$R_{(c-a)} = 1/(1/R_{r(c-a)} + 1/R_{c(c-a)})$$

Radiation

$$R_{r(c-a)} = 1/h_{r(c-a)} = 1/\varepsilon_c \sigma (T_a^2 + T_c^2)(T_a + T_c)$$

Convection

$$R_{c(c-a)} = 1/h_{c(c-a)} = 1/h_w$$

Plate to cover

$$R_{(p-c)} = 1/(1/R_{r(p-c)} + 1/R_{c(p-c)})$$

Radiation

$$R_{r(p-c)} = 1/h_{r(p-c)} = 1/[\sigma(T_c^2 + T_p^2)(T_c + T_p)/(1/\varepsilon_c + 1/\varepsilon_p - 1)]$$

Convection

$$R_{c(p-c)} = 1/h_{c(p-c)} = 1/h_c$$

Assume T_c between ambient and absorber plate temperature

Assumption validation !

$$T_c = T_p - U_{top}(T_p - T_a) \times R_{(p-c)}$$