

# **Chapter 3**

## **Evaluating Properties**

# Learning Outcomes

- ▶ Demonstrate understanding of key concepts . . . including phase and pure substance, state principle for simple compressible systems,  $p$ - $v$ - $T$  surface, saturation temperature and saturation pressure, two-phase liquid-vapor mixture, quality, enthalpy, and specific heats.
- ▶ Apply the closed system energy balance with property data.

# Learning Outcomes, cont.

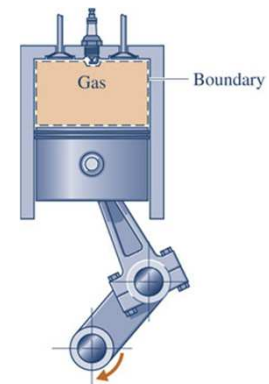
- ▶ Sketch  $T-v$ ,  $p-v$ , and phase diagrams, and locate states on these diagrams.
- ▶ Retrieve property data from Tables A-1 through A-23.
- ▶ Apply the ideal gas model for thermodynamic analysis, including determining when use of the model is warranted.

# Phase

- ▶ A quantity of matter that is **homogeneous throughout** in both **chemical composition** and **physical structure**.
- ▶ **Homogeneity** in physical structure means that the matter is all *solid*, or all *liquid*, or all *vapor (gas)*.
- ▶ **Examples:**
  - ▶ The **air** we breathe is a **gas phase** consisting of a mixture of different gases.
  - ▶ **Drinking water with ice cubes** contains **two phases of water**: liquid and solid.
  - ▶ **Vinegar and olive oil salad dressing** contains **two different liquid phases**.

# Pure Substance

- ▶ A substance that is **uniform** and **invariable in chemical composition**.
- ▶ A pure substance **can exist in more than one phase**, but its **chemical composition must be the same in each phase**.
- ▶ **Examples:**
  - ▶ **Drinking water with ice cubes** can be regarded as a **pure substance** because **each phase has the same composition**.
  - ▶ A **fuel-air mixture** in the cylinder of an automobile engine can be regarded as a **pure substance** until ignition occurs.



# State Principle for Simple Compressible Systems

- ▶ Systems of commonly encountered pure substances are called **simple compressible systems**. These substances include those in appendix tables A-2 through A-18, A-22, and A-23.
- ▶ The **intensive state of a simple compressible system at equilibrium is described by its intensive properties**, including **temperature, pressure, specific volume, density, specific internal energy, and specific enthalpy**.
- ▶ Properties such as **velocity** and **elevation** are excluded because their values depend on arbitrary datum choices, such as zero values at the surface of the earth. For the **state principle**, these properties are not relevant.

# State Principle for Simple Compressible Systems

- ▶ Not all of the relevant intensive properties are independent.
  - ▶ Some are related by definitions – **for example**, density is  $1/v$  and specific enthalpy is  $u + pv$  (Eq. 3.4).
  - ▶ Others are related through expressions developed from experimental data.
  - ▶ Some intensive properties may be independent in a single phase, but become dependent when there is more than one phase present.

## State Principle for Simple Compressible Systems

- ▶ For a simple compressible system, **values for *any two* independent intensive properties determine the values of *all other* intensive properties.** This is the **state principle for simple compressible systems.**
- ▶ Among alternative sets of two independent intensive properties,  **$(T, v)$  and  $(p, v)$**  are frequently convenient. We soon show that **temperature and pressure are not always an independent set.**

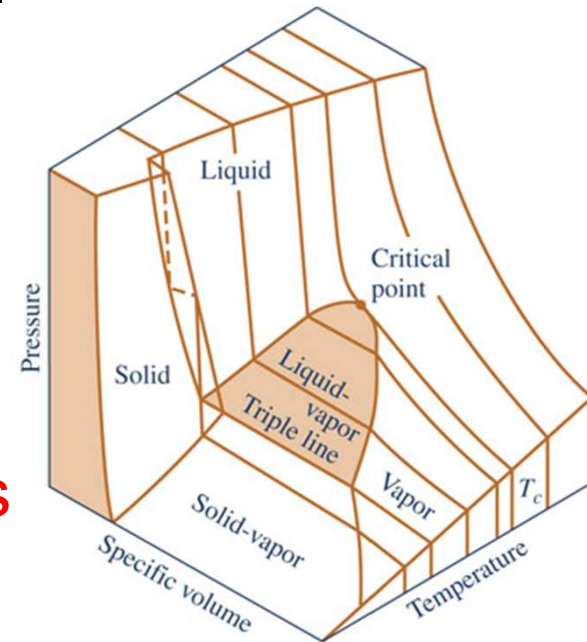


# $p-v-T$ Surface

- ▶ For pure, simple compressible systems, pressure can be determined as a function of temperature and specific volume:

$$p = p(T, v)$$

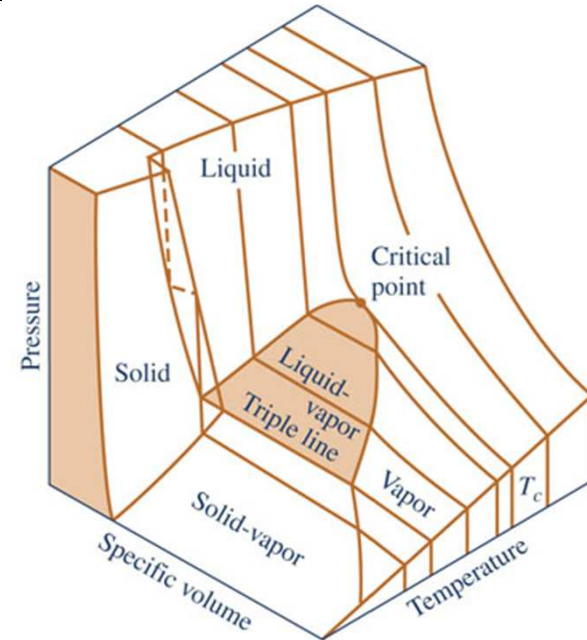
The graph of this relation for water is indicated by the  $p-v-T$  surface shown.



- ▶ **Single-phase regions** on the surface include *solid*, *liquid*, and *vapor*.
- ▶ **Two-phase regions** are located between single-phase regions, where two phases exist in equilibrium: *liquid-vapor*, *solid-vapor*, *solid-liquid*.

# $p$ - $v$ - $T$ Surface

- ▶ The dome-shaped region composed of the two-phase liquid-vapor states is called the **vapor dome**.
- ▶ A state at which a phase change begins or ends is called a **saturation state**. Lines bordering the vapor dome are called the **saturated liquid** and **saturated vapor lines**.
- ▶ At the top of the dome, where saturated liquid and saturated vapor lines meet, is the **critical point**.
  - ▶ **Critical temperature** ( $T_c$ ) is the maximum temperature at which liquid and vapor phases can coexist in equilibrium.
  - ▶ **Critical pressure** ( $p_c$ ) is pressure at the critical point.



# Projections of the $p$ - $v$ - $T$ Surface

▶ Projection of the  $p$ - $v$ - $T$  surface onto the pressure-temperature plane is a **phase diagram**.

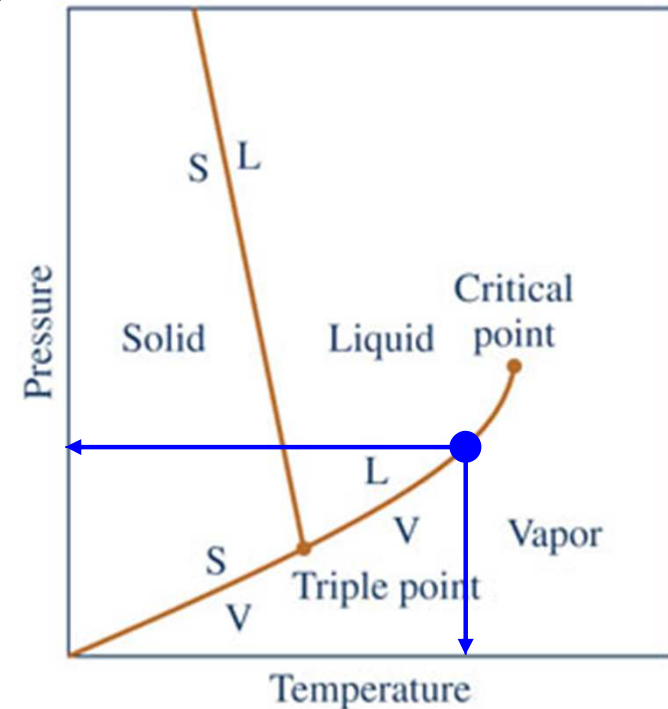
▶ **Saturation temperature**

designates the temperature at which a phase change takes place at a given pressure.

▶ **Saturation pressure**

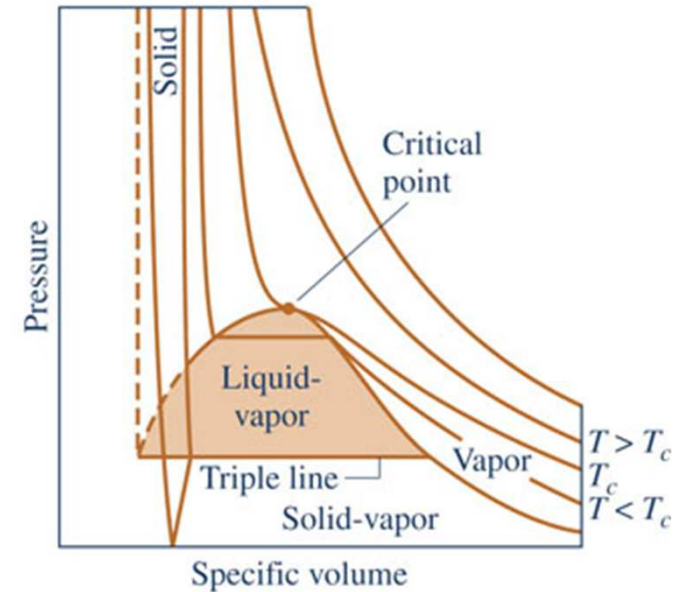
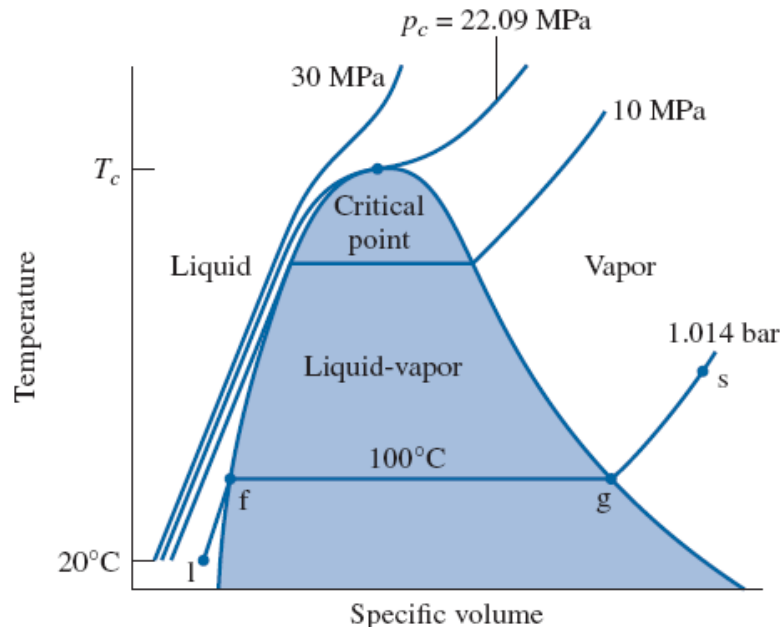
designates the pressure at which a phase change takes place at a given temperature.

▶ Within **two-phase regions** pressure and temperature are **not** independent.



# Projections of the $p$ - $v$ - $T$ Surface

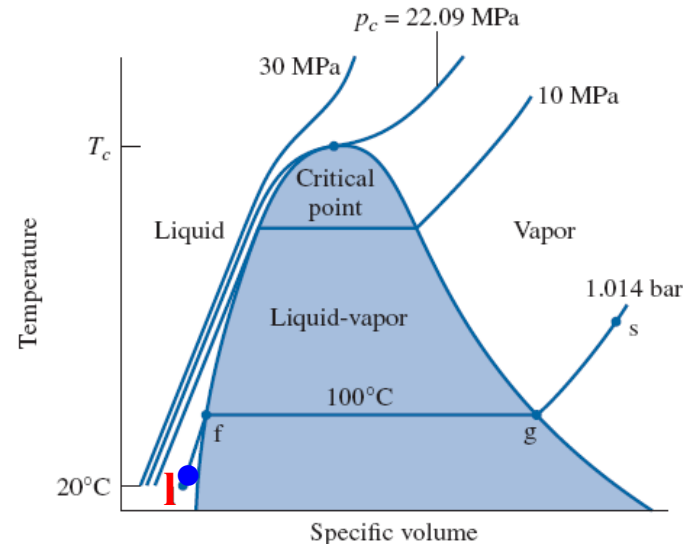
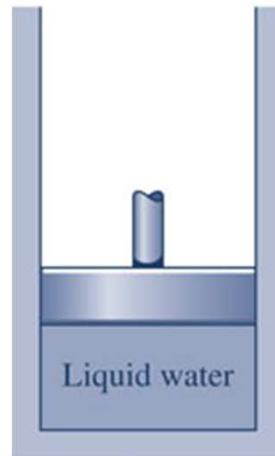
- ▶ Projection of the  $p$ - $v$ - $T$  surface onto the pressure-specific volume plane results in a  **$p$ - $v$  diagram**.



- ▶ Projection of the  $p$ - $v$ - $T$  surface onto the temperature-specific volume plane results in a  **$T$ - $v$  diagram**.

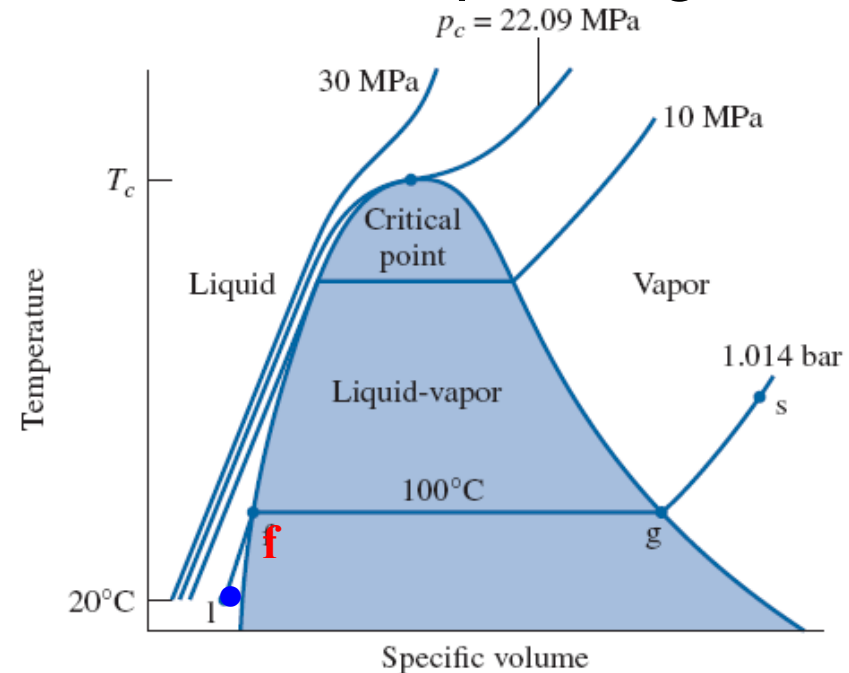
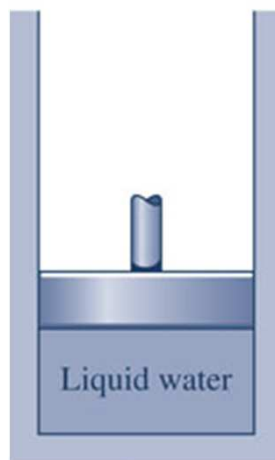
# Phase Change

- ▶ Consider a closed system consisting of a unit mass of **liquid water** at 20°C contained within a piston-cylinder assembly.
- ▶ This state is represented by **I** (highlighted by the **blue dot**).
- ▶ Liquid states such as this, where temperature is lower than the saturation temperature corresponding to the pressure at the state, are called **compressed liquid** states.



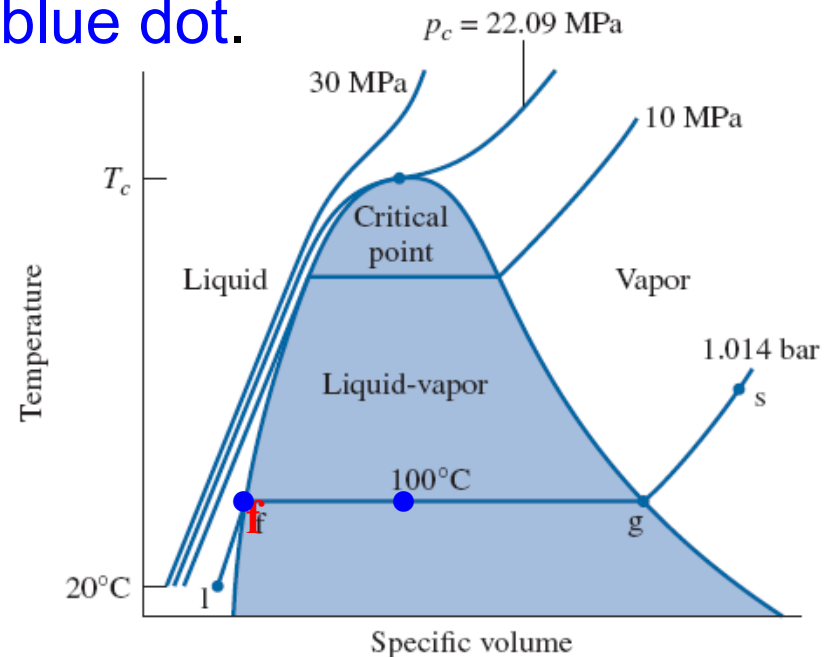
# Saturated Liquid

- ▶ As the **system is heated** at constant pressure, the **temperature increases considerably while the specific volume increases slightly**.
- ▶ Eventually, the system is brought to the state represented by **f** (highlighted by the **blue dot**).
- ▶ This is the **saturated liquid** state corresponding to the specified pressure.



# Two-Phase Liquid-Vapor Mixture

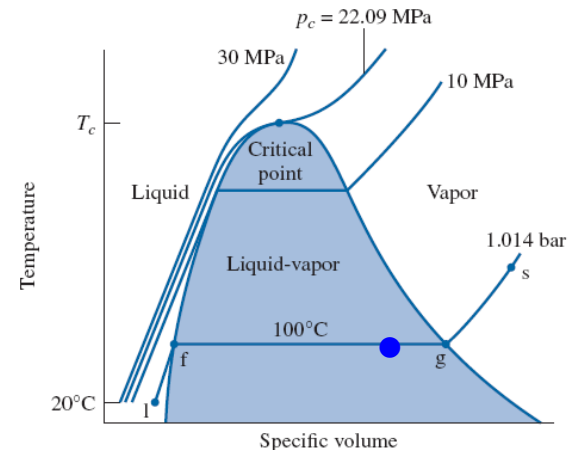
- ▶ When the system is at the saturated liquid state, **additional heat transfer** at fixed pressure results in the **formation of vapor without change in temperature** but with a **considerable increase in specific volume** as shown by movement of the **blue dot**.
- ▶ With **additional heating** at fixed pressure, **more vapor is formed** and **specific volume increases further** as shown by additional movement of the **blue dot**.
- ▶ At these states, the system now consists of a **two-phase liquid-vapor mixture**.



# Two-Phase Liquid-Vapor Mixture

- ▶ When a mixture of liquid and vapor exists in equilibrium, the liquid phase is a saturated liquid and the vapor phase is a saturated vapor.
- ▶ For a **two-phase liquid-vapor mixture**, the ratio of the mass of vapor present to the total mass of the mixture is its **quality**,  $x$ .
- ▶ The **value of quality** ranges from 0 to 1.
- ▶ At **saturated liquid states**,  $x = 0$ .

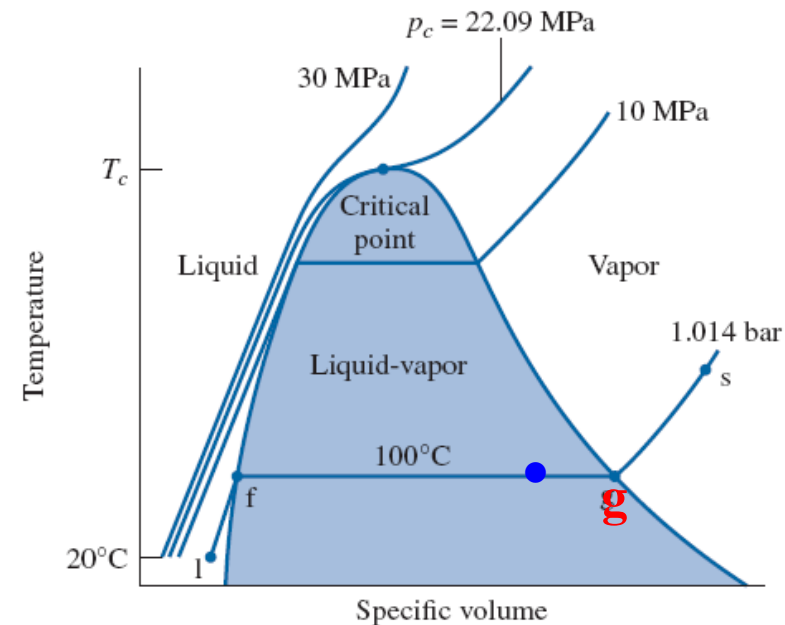
$$x = \frac{m_{\text{vapor}}}{m_{\text{liquid}} + m_{\text{vapor}}}$$





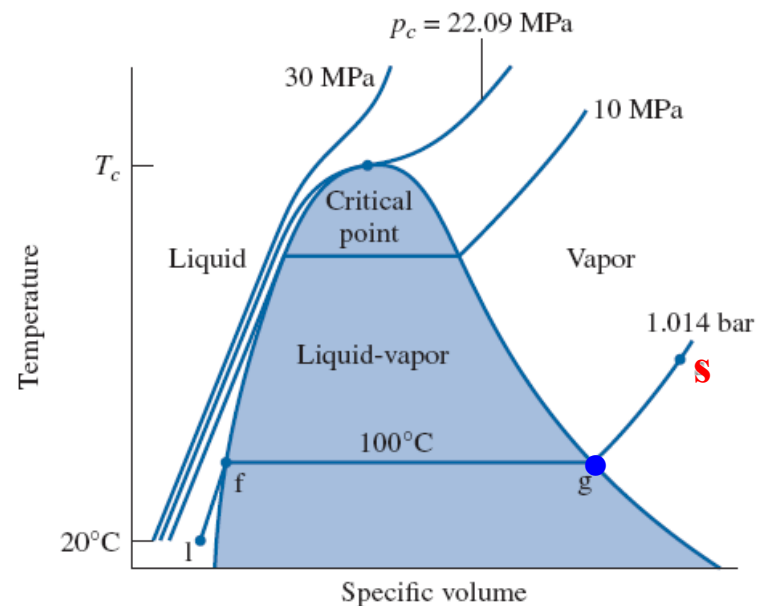
# Saturated Vapor

- ▶ If the system is heated further until the last bit of liquid has vaporized it is brought to the **saturated vapor** state.
- ▶ This state is represented by **g** (highlighted by the blue dot).
- ▶ At **saturated vapor states**,  $x = 1$ .



# Superheated Vapor

- ▶ When the system is at the saturated vapor state, **further heating** at fixed pressure results in **increases in both temperature and specific volume**.
- ▶ This state is represented by **s** (highlighted by the **blue dot**).
- ▶ Vapor states such as this, where temperature is higher than the saturation temperature corresponding to the pressure at the state, are called **superheated vapor** states.

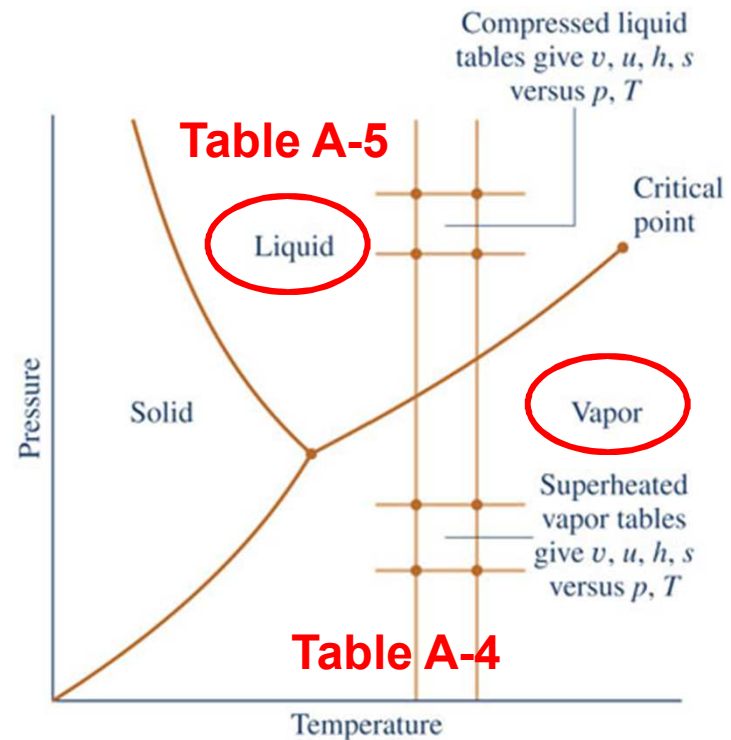


# Steam Tables

- ▶ Tables of properties for different substances are frequently set up in the same general format. The tables for water, called the **steam tables**, provide an example of this format. The steam tables are in appendix **tables A-2 through A-5**.
  - ▶ Table A-4 applies to water as a **superheated vapor**.
  - ▶ Table A-5 applies to **compressed liquid** water.
  - ▶ Tables A-2 and A-3 apply to the **two-phase, liquid-vapor mixture** of water.

# Single-Phase Regions

- ▶ Since **pressure and temperature are independent properties** in the **single-phase liquid and vapor regions**, they can be used to fix the state in these regions.
- ▶ **Tables A-4** (Superheated Water Vapor) and **A-5** (Compressed Liquid Water) **provide several properties as functions of pressure and temperature**, as considered next.



# Single-Phase Regions

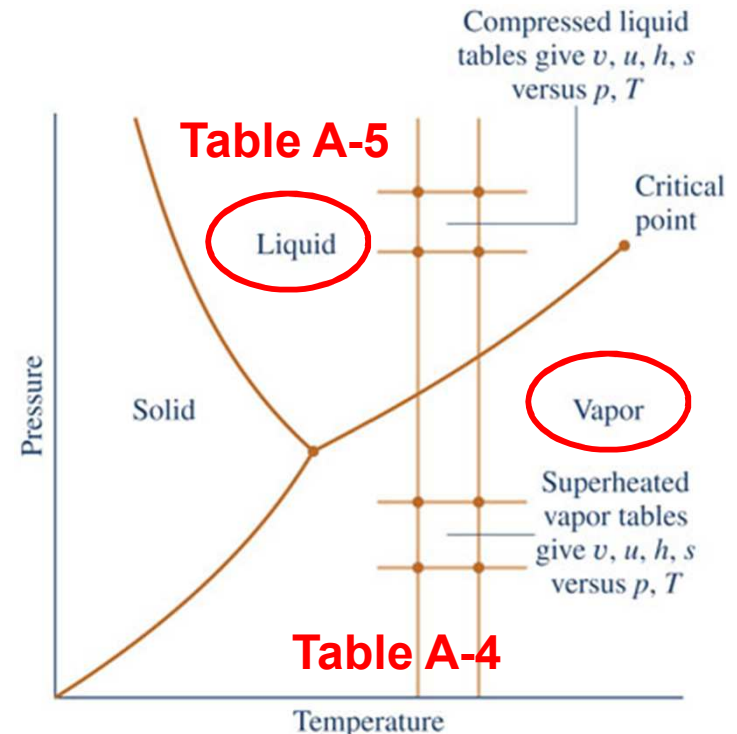
Properties tabulated in **Tables A-4** and **A-5** include

- ▶ Temperature ( $T$ )
- ▶ Pressure ( $p$ )
- ▶ Specific volume ( $v$ )
- ▶ Specific internal energy ( $u$ )
- ▶ Specific enthalpy ( $h$ ), which is a sum of terms that often appears in thermodynamic analysis:

$$h = u + pv$$

**Enthalpy is a property** because it is defined in terms of properties; physical significance is associated with it in Chapter 4.

- ▶ Specific entropy ( $s$ ), an intensive property developed in Chapter 6



# Single-Phase Regions

► **Example:** Properties associated with **superheated water vapor** at **10 MPa** and **400°C** are found in **Table A-4**.

►  $v = 0.02641 \text{ m}^3/\text{kg}$

►  $h = 3096.5 \text{ kJ/kg}$

►  $u = 2832.4 \text{ kJ/kg}$

►  $s = 6.2120 \text{ kJ/kg}\cdot\text{K}$

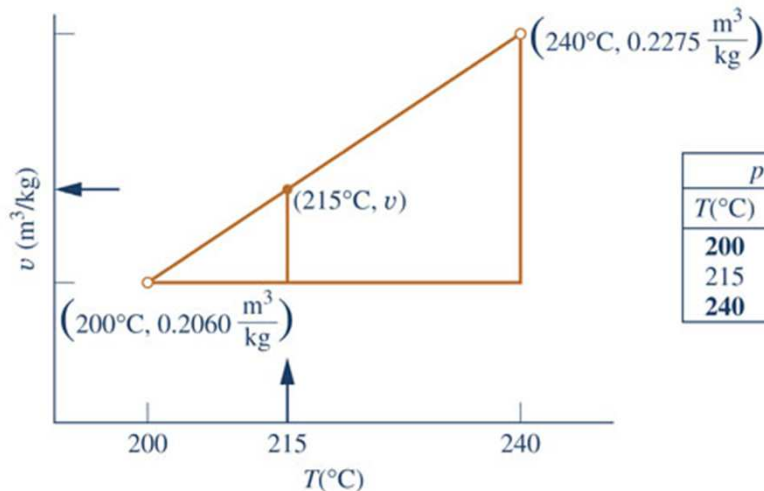
**Table A-4**

$T$ °C	$v$ m <sup>3</sup> /kg	$u$ kJ/kg	$h$ kJ/kg	$s$ kJ/kg·K	$v$ m <sup>3</sup> /kg	$u$ kJ/kg	$h$ kJ/kg	$s$ kJ/kg·K
	$p = 80 \text{ bar} = 8.0 \text{ MPa}$ ( $T_{\text{sat}} = 295.06^\circ\text{C}$ )				$p = 100 \text{ bar} = 10.0 \text{ MPa}$ ( $T_{\text{sat}} = 311.06^\circ\text{C}$ )			
Sat.	0.02352	2569.8	2758.0	5.7432	0.01803	2544.4	2724.7	5.6141
320	0.02682	2662.7	2877.2	5.9489	0.01925	2588.8	2781.3	5.7103
360	0.03089	2772.7	3019.8	6.1819	0.02331	2729.1	2962.1	6.0060
400	0.03432	2863.8	3138.3	6.3634	0.02641	2832.4	3096.5	6.2120
440	0.03742	2946.7	3246.1	6.5190	0.02911	2922.1	3213.2	6.3805
480	0.04034	3025.7	3348.4	6.6586	0.03160	3005.4	3321.4	6.5282

# Linear Interpolation

- ▶ When a state does not fall exactly on the grid of values provided by property tables, **linear interpolation** between adjacent entries is used.
- ▶ **Example:** Specific volume ( $v$ ) associated with **superheated water vapor** at **10 bar** and **215°C** is found by **linear interpolation** between adjacent entries in **Table A-4**.

$$\text{slope} = \frac{(0.2275 - 0.2060) \text{ m}^3/\text{kg}}{(240 - 200)^\circ\text{C}} = \frac{(v - 0.2060) \text{ m}^3/\text{kg}}{(215 - 200)^\circ\text{C}} \rightarrow v = 0.2141 \text{ m}^3/\text{kg}$$



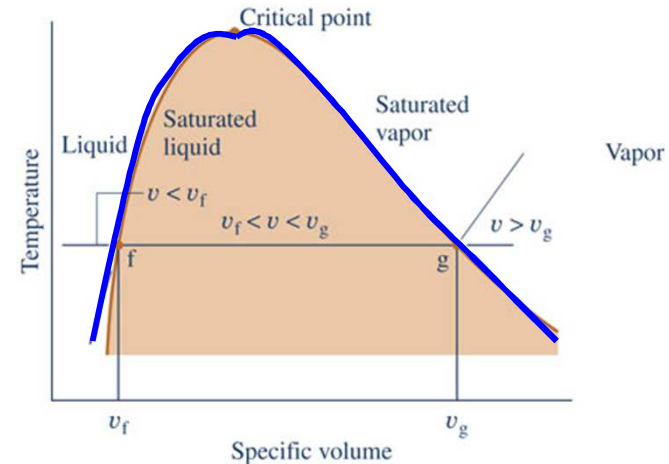
$p = 10 \text{ bar}$	
$T(^{\circ}\text{C})$	$v \text{ (m}^3/\text{kg)}$
200	0.2060
215	$v = ?$
240	0.2275

**Table A-4**

$T$ $^{\circ}\text{C}$	$v$ $\text{m}^3/\text{kg}$	$u$ $\text{kJ/kg}$	$h$ $\text{kJ/kg}$	$s$ $\text{kJ/kg}\cdot\text{K}$
$p = 10 \text{ bar} = 1.0 \text{ MPa}$ ( $T_{\text{sat}} = 179.91^{\circ}\text{C}$ )				
Sat.	0.1944	2583.6	2778.1	6.5865
200	0.2060	2621.9	2827.9	6.6940
240	0.2275	2692.9	2920.4	6.8817

# Two-Phase Liquid-Vapor Region

- ▶ **Tables A-2/** (Temperature Table) and **A-3** (Pressure Table) **provide**
  - ▶ saturated liquid (**f**) data
  - ▶ saturated vapor (**g**) data



**Table note:** For saturated liquid specific volume, the table heading is  $v_f \times 10^3$ .  
**At 8°C,  $v_f \times 10^3 = 1.002 \rightarrow v_f = 1.002/10^3 = 1.002 \times 10^{-3}$ .**

Table A-2		Specific Volume m <sup>3</sup> /kg		Internal Energy kJ/kg		Enthalpy kJ/kg			Entropy kJ/kg·K		Temp °C
		Sat. Liquid $v_f \times 10^3$	Sat. Vapor $v_g$	Sat. Liquid $u_f$	Sat. Vapor $u_g$	Sat. Liquid $h_f$	Evap. $h_{fg}$	Sat. Vapor $h_g$	Sat. Liquid $s_f$	Sat. Vapor $s_g$	
.01	0.00611	1.0002	206.136	0.00	2375.3	0.01	2501.3	2501.4	0.0000	9.1562	.01
4	0.00813	1.0001	157.232	16.77	2380.9	16.78	2491.9	2508.7	0.0610	9.0514	4
5	0.00872	1.0001	147.120	20.97	2382.3	20.98	2489.6	2510.6	0.0761	9.0257	5
6	0.00935	1.0001	137.734	25.19	2383.6	25.20	2487.2	2512.4	0.0912	9.0003	6
8	0.01072	1.0002	120.917	33.59	2386.4	33.60	2482.5	2516.1	0.1212	8.9501	8



# Two-Phase Liquid-Vapor Region

▶ The **specific volume of a two-phase liquid-vapor mixture** can be determined by using the saturation tables and quality,  $x$ .

▶ The **total volume of the mixture** is the sum of the volumes of the liquid and vapor phases:

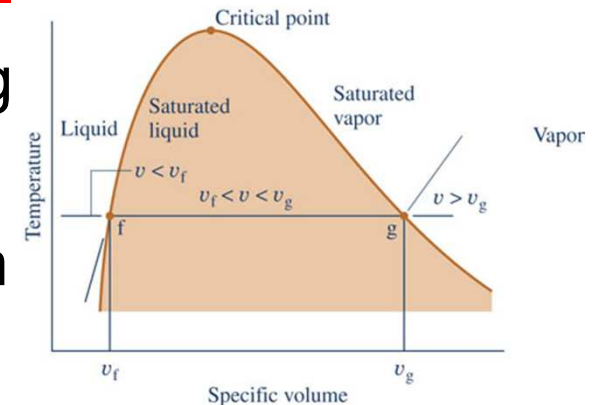
$$V = V_{\text{liq}} + V_{\text{vap}}$$

▶ Dividing by the total mass of the mixture,  $m$ , **an average specific volume for the mixture** is:

$$v = \frac{V}{m} = \frac{V_{\text{liq}}}{m} + \frac{V_{\text{vap}}}{m}$$

▶ With  $V_{\text{liq}} = m_{\text{liq}}v_f$ ,  $V_{\text{vap}} = m_{\text{vap}}v_g$ ,  $m_{\text{vap}}/m = x$ , and  $m_{\text{liq}}/m = 1 - x$ :

$$v = (1 - x)v_f + xv_g = v_f + x(v_g - v_f) \quad \text{(Eq. 3.2)}$$



## Two-Phase Liquid-Vapor Region

- ▶ Since **pressure and temperature are NOT independent properties** in the **two-phase liquid-vapor region**, they cannot be used to fix the state in this region.
- ▶ The property, **quality ( $x$ )**, defined only in the two-phase liquid-vapor region, **and either temperature or pressure can be used to fix the state in this region.**

$$v = (1 - x)v_f + xv_g = v_f + x(v_g - v_f) \quad \text{(Eq. 3.2)}$$

$$u = (1 - x)u_f + xu_g = u_f + x(u_g - u_f) \quad \text{(Eq. 3.6)}$$

$$h = (1 - x)h_f + xh_g = h_f + x(h_g - h_f) \quad \text{(Eq. 3.7)}$$

# Two-Phase Liquid-Vapor Region

► **Example:** A system consists of a **two-phase liquid-vapor mixture of water** at **6°C** and a **quality of 0.4**. Determine the specific volume, in  $\text{m}^3/\text{kg}$ , of the mixture.

► **Solution:** Apply **Eq. 3.2**,  $v = v_f + x(v_g - v_f)$

Substituting values from **Table 2**:  $v_f = 1.001 \times 10^{-3} \text{ m}^3/\text{kg}$  and  $v_g = 137.734 \text{ m}^3/\text{kg}$ :

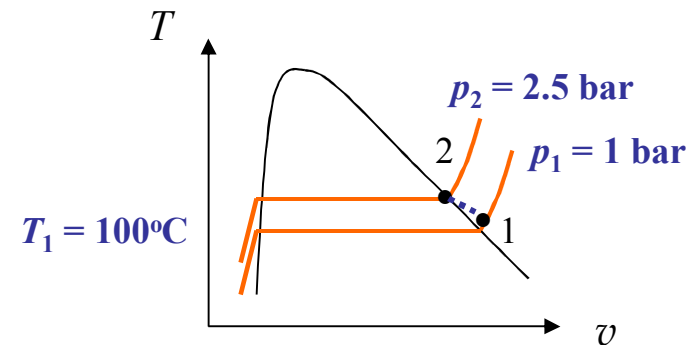
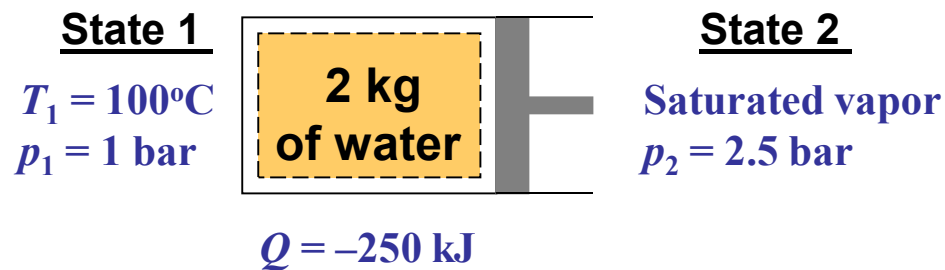
$$v = 1.001 \times 10^{-3} \text{ m}^3/\text{kg} + 0.4(137.734 - 1.001 \times 10^{-3}) \text{ m}^3/\text{kg}$$

$$v = 55.094 \text{ m}^3/\text{kg}$$

Table A-2		Specific Volume $\text{m}^3/\text{kg}$		Internal Energy $\text{kJ}/\text{kg}$		Enthalpy $\text{kJ}/\text{kg}$			Entropy $\text{kJ}/\text{kg}\cdot\text{K}$		Temp $^{\circ}\text{C}$
Temp $^{\circ}\text{C}$	Press. bar	Sat. Liquid $v_f \times 10^3$	Sat. Vapor $v_g$	Sat. Liquid $u_f$	Sat. Vapor $u_g$	Sat. Liquid $h_f$	Evap. $h_{fg}$	Sat. Vapor $h_g$	Sat. Liquid $s_f$	Sat. Vapor $s_g$	
.01	0.00611	1.0002	206.136	0.00	2375.3	0.01	2501.3	2501.4	0.0000	9.1562	.01
4	0.00813	1.0001	157.232	16.77	2380.9	16.78	2491.9	2508.7	0.0610	9.0514	4
5	0.00872	1.0001	147.120	20.97	2382.3	20.98	2489.6	2510.6	0.0761	9.0257	5
6	0.00935	1.0001	137.734	25.19	2383.6	25.20	2487.2	2512.4	0.0912	9.0003	6
8	0.01072	1.0002	120.917	33.59	2386.4	33.60	2482.5	2516.1	0.1212	8.9501	8

# Property Data Use in the Closed System Energy Balance

**Example:** A piston-cylinder assembly contains **2 kg** of water at **100°C** and **1 bar**. The water is compressed to a **saturated vapor** state where the pressure is **2.5 bar**. During compression, there is a heat transfer of energy from the water to its surroundings having a magnitude of **250 kJ**. Neglecting changes in kinetic energy and potential energy, determine the work, in kJ, for the process of the water.



## Property Data Use in the Closed System Energy Balance

**Solution:** An energy balance for the closed system is

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$$

where the kinetic and potential energy changes are neglected.

**Thus**

$$W = Q - m(u_2 - u_1)$$

**State 1** is in the **superheated vapor** region and is fixed by  $p_1 = 1$  bar and  $T_1 = 100^\circ\text{C}$ . From **Table A-4**,  $u_1 = 2506.7$  kJ/kg.

**State 2** is **saturated vapor** at  $p_2 = 2.5$  bar. From **Table A-3**,  $u_2 = u_g = 2537.2$  kJ/kg.

$$W = -250 \text{ kJ} - (2 \text{ kg})(2537.2 - 2506.7) \text{ kJ/kg} = -311 \text{ kJ}$$

The negative sign indicates work is done **on** the system as expected for a compression process.

# Specific Heats

- ▶ Three properties related to specific internal energy and specific enthalpy having important applications are the **specific heats**  $c_v$  and  $c_p$  and the **specific heat ratio**  $k$ .

$$c_v = \left( \frac{\partial u}{\partial T} \right)_v$$

(Eq. 3.8)

$$c_p = \left( \frac{\partial h}{\partial T} \right)_p$$

(Eq. 3.9)

$$k = \frac{c_p}{c_v}$$

(Eq. 3.10)

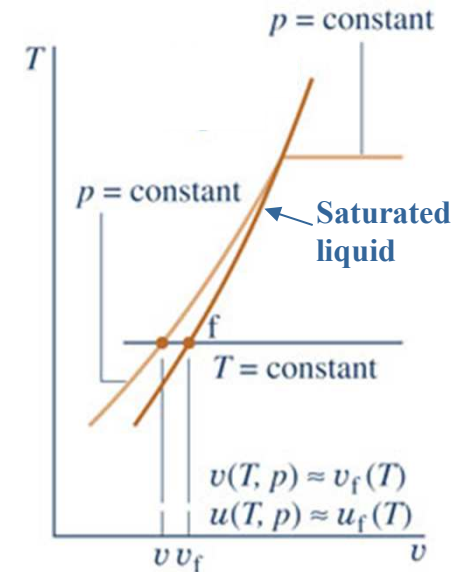
- ▶ In general,  $c_v$  is a function of  $v$  and  $T$  (or  $p$  and  $T$ ), and  $c_p$  depends on both  $p$  and  $T$  (or  $v$  and  $T$ ).
- ▶ Specific heat data are provided in **Fig 3.9** and **Tables A-19** through **A-21**.
- ▶ Although  $c_v$  and  $c_p$  are referred to as **specific heats**, there is no general relationship between them and the heat transfer term of the energy balance denoted by  $Q$ .

# Property Approximations for Liquids

- ▶ **Approximate values** for  $v$ ,  $u$ , and  $h$  at liquid states can be obtained using **saturated liquid data**.
- ▶ Since the **values of  $v$  and  $u$  for liquids change very little with pressure at a fixed temperature**, **Eqs. 3.11** and **3.12** can be used to approximate their values.

$$v(T, p) \approx v_f(T) \quad (\text{Eq. 3.11})$$

$$u(T, p) \approx u_f(T) \quad (\text{Eq. 3.12})$$



- ▶ An **approximate value** for  $h$  at liquid states can be obtained using **Eqs. 3.11** and **3.12** in the definition  $h = u + pv$ :  $h(T, p) \approx u_f(T) + pv_f(T)$  or alternatively

$$h(T, p) \approx h_f(T) + \underline{v_f(T)[p - p_{\text{sat}}(T)]} \quad (\text{Eq. 3.13})$$

**where**  $p_{\text{sat}}$  denotes the saturation pressure at the given temperature

- ▶ When the underlined term in **Eq. 3.13** is small

$$h(T, p) \approx h_f(T) \quad (\text{Eq. 3.14})$$

# Incompressible Substance Model

▶ For a substance modeled as *incompressible*

▶  $v = \text{constant}$

▶  $u = u(T)$

▶ For a substance modeled as incompressible,  $c_p = c_v$ ; the common specific heat value is represented by  $c$ .

▶ For a substance modeled as incompressible with constant  $c$ :

$$u_2 - u_1 = c(T_2 - T_1)$$

(Eq. 3.20a)

$$h_2 - h_1 = c(T_2 - T_1) + \underline{v(p_2 - p_1)}$$

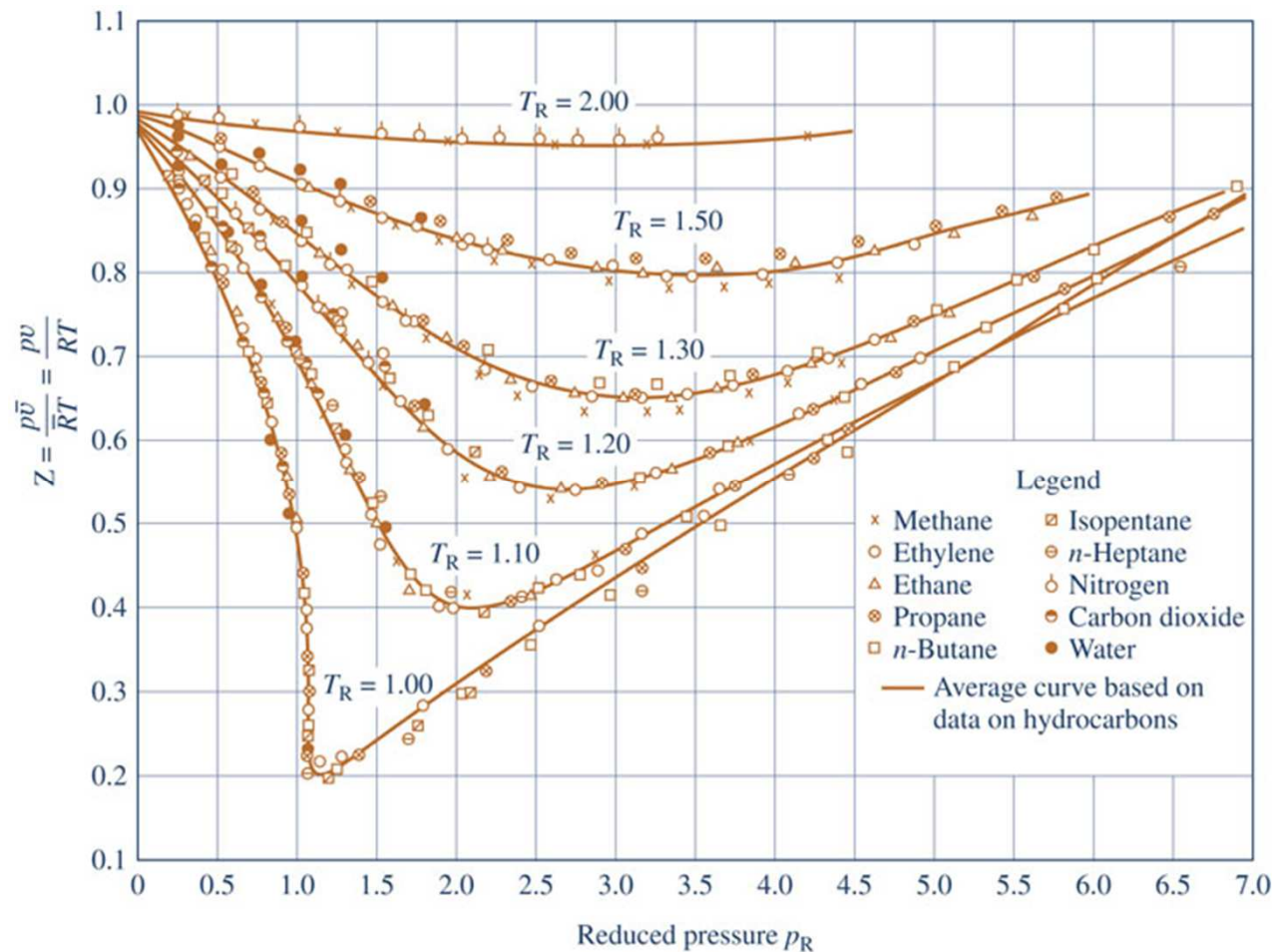
(Eq. 3.20b)

▶ In **Eq. 3.20b**, the contribution of the underlined term is often small enough to be ignored.



# Generalized Compressibility Chart

- The  $p\text{-}\bar{v}\text{-}T$  relation for 10 common gases is shown in the **generalized compressibility chart**.



# Generalized Compressibility Chart

► In this chart, the **compressibility factor**,  $Z$ , is plotted versus the reduced pressure,  $p_R$ , and reduced temperature  $T_R$ , where

$$Z = \frac{p\bar{v}}{RT}$$

(Eq. 3.23)

$$p_R = p/p_c$$

(Eq. 3.27)

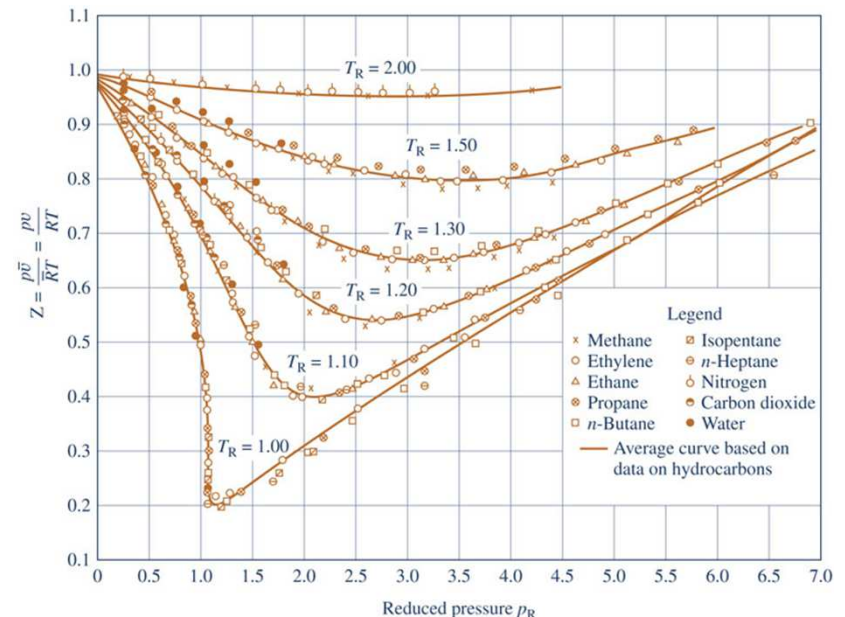
$$T_R = T/T_c$$

(Eq. 3.28)

$\bar{R}$  is the **universal gas constant**

$$\bar{R} = \begin{cases} 8.314 \text{ kJ/kmol}\cdot\text{K} \\ 1.986 \text{ Btu/lbmol}\cdot\text{°R} \\ 1545 \text{ ft}\cdot\text{lbf/lbmol}\cdot\text{°R} \end{cases} \quad (\text{Eq. 3.22})$$

The symbols  $p_c$  and  $T_c$  denote the temperature and pressure at the critical point for the particular gas under consideration. These values are obtained from **Tables A-1**.



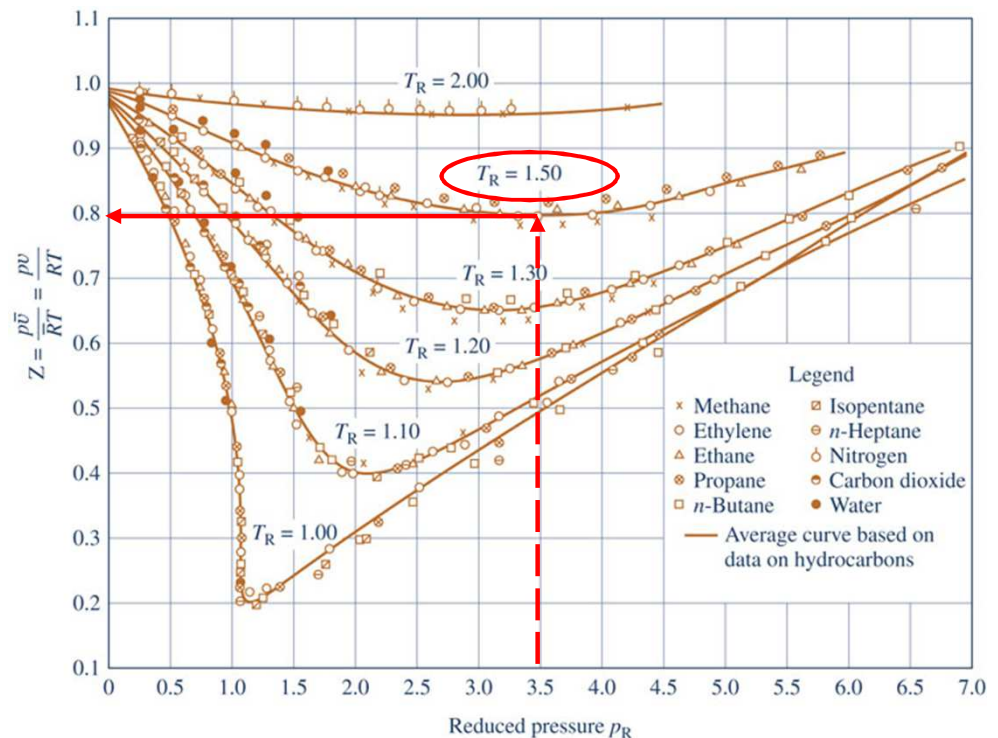
# Generalized Compressibility Chart

- ▶ When  $p$ ,  $p_c$ ,  $T$ ,  $T_c$ ,  $\bar{v}$ , and  $\bar{R}$  are used in consistent units,  $Z$ ,  $p_R$ , and  $T_R$  are numerical values without units.
- ▶ **Example:** For air at 200 K, 132 bar,  $T_R = 200 \text{ K}/133 \text{ K} = 1.5$ ,  $p_R = 132 \text{ bar}/37.7 \text{ bar} = 3.5$  where  $T_c$  and  $p_c$  for air are from **Table A-1**. With these  $T_R$ ,  $p_R$  values, the generalized compressibility chart gives  $Z = 0.8$ .
- ▶ With  $\bar{v} = Mv$ , an alternative form of the compressibility factor is

$$Z = \frac{p\bar{v}}{RT} \quad (\text{Eq. 3.24})$$

where

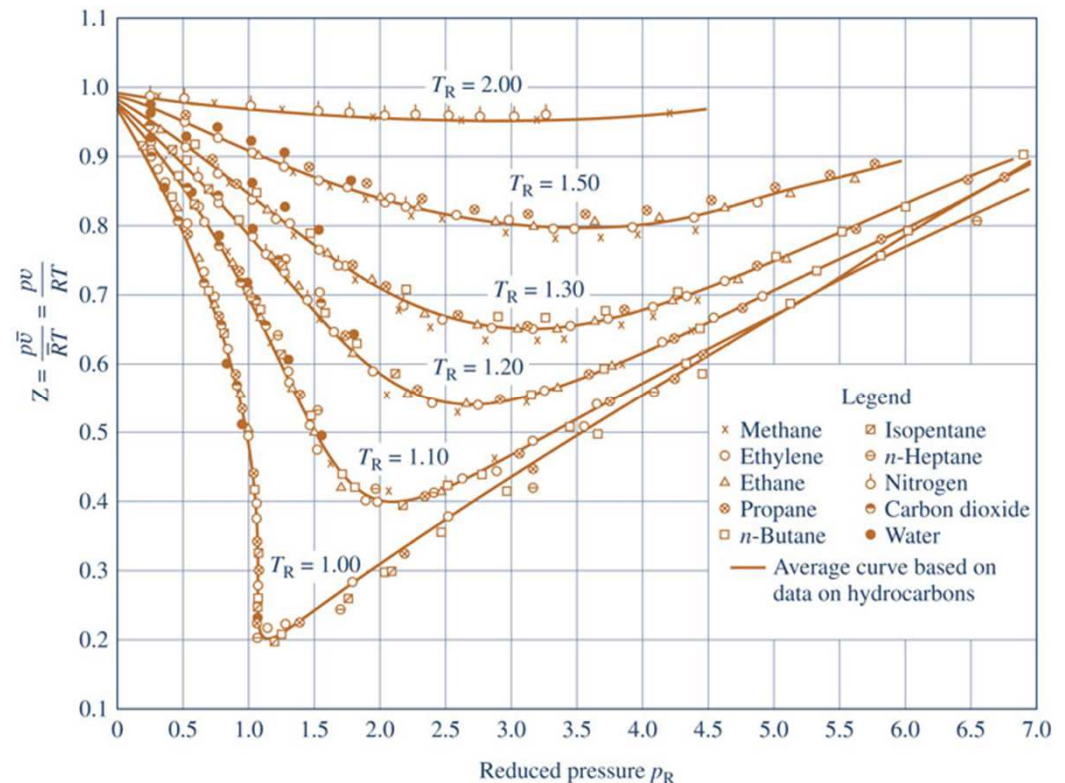
$$R = \frac{\bar{R}}{M} \quad (\text{Eq. 3.25})$$



# Studying the Generalized Compressibility Chart

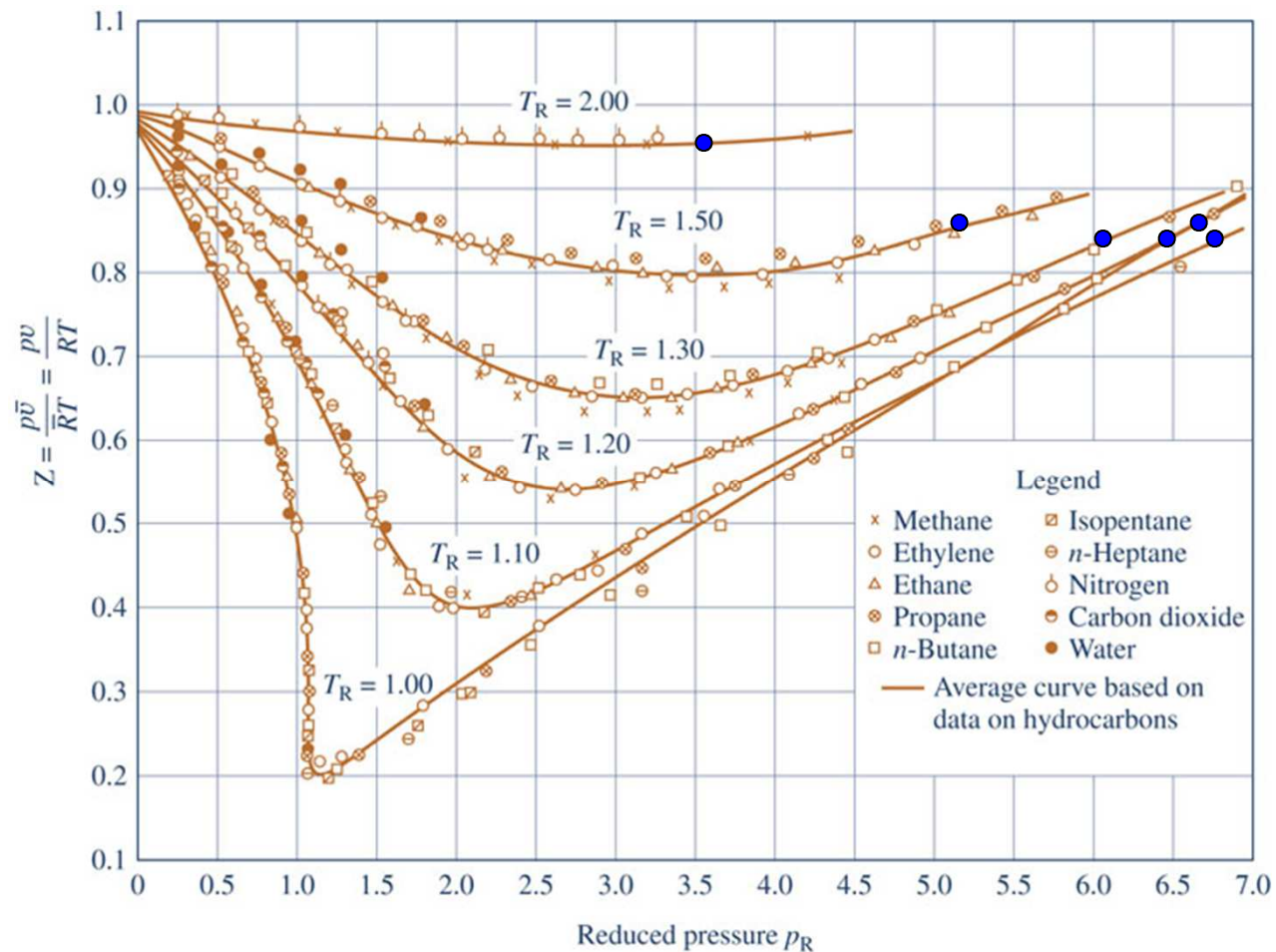
► The solid lines labeled with  $T_R$  values represent best fits to experimental data. For the 10 different gases represented there is little scatter in data about these lines.

► At the lowest reduced temperature value shown,  $T_R = 1.0$ , the compressibility factor varies between 0.2 and 1.0. Less variation is observed as  $T_R$  takes higher values.



# Studying the Generalized Compressibility Chart

- ▶ For each specified value of  $T_R$ , the **compressibility factor approaches a value of 1.0 as  $p_R$  approaches zero.**



## Studying the Generalized Compressibility Chart

- ▶ Low values of  $p_R$ , where  $Z \approx 1$ , do not necessarily correspond to a range of low absolute pressures.
- ▶ For instance, **if  $p_R = 0.05$ , then  $p = 0.05p_c$** . With  $p_c$  values from **Table A-1**

<b>Water vapor</b>	$p_c = 220.9 \text{ bar} \rightarrow p = 11 \text{ bar}$
<b>Ammonia</b>	$p_c = 112.8 \text{ bar} \rightarrow p = 5.6 \text{ bar}$
<b>Carbon dioxide</b>	$p_c = 73.9 \text{ bar} \rightarrow p = 3.7 \text{ bar}$
<b>Air</b>	$p_c = 37.7 \text{ bar} \rightarrow p = 1.9 \text{ bar}$

- ▶ These pressure values range from 1.9 to 11 bar, which in engineering practice are not normally considered as low pressures.

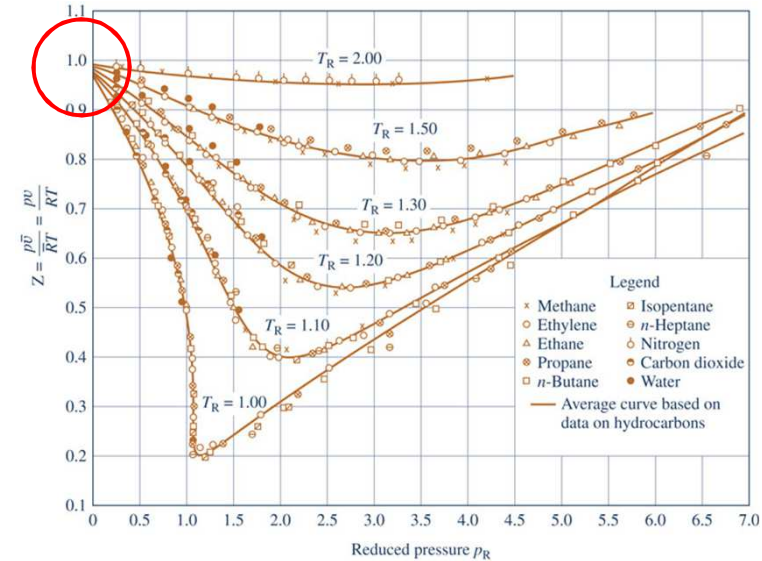
# Introducing the Ideal Gas Model

► To recap, the generalized compressibility chart shows that at states where the pressure  $p$  is small relative to the critical pressure  $p_c$  (where  $p_R$  is small), the compressibility factor  $Z$  is approximately 1.

► At such states, it can be assumed with reasonable accuracy that  $Z = 1$ . Then

$$pv = RT$$

(Eq. 3.32)



# Introducing the Ideal Gas Model

▶ Three alternative forms of **Eq. 3.32** can be derived as follows:

▶ With  $v = V/m$ , **Eq. 3.32** gives

$$pV = mRT \quad (\text{Eq. 3.33})$$

▶ With  $v = \bar{v}/M$  and  $R = \bar{R}/M$ , **Eq. 3.32** gives

$$p\bar{v} = \bar{R}T \quad (\text{Eq. 3.34})$$

▶ Finally, with  $\bar{v} = V/n$ , **Eq. 3.34** gives

$$pV = n\bar{R}T \quad (\text{Eq. 3.35})$$



# Introducing the Ideal Gas Model

- ▶ Investigation of **gas behavior at states where Eqs. 3.32-3.35 are applicable** indicates that the **specific internal energy depends primarily on temperature**. Accordingly, at such states, it can be assumed with reasonable accuracy that it depends on temperature alone:

$$u = u(T) \quad (\text{Eq. 3.36})$$

- ▶ With **Eqs. 3.32 and 3.36**, the specific enthalpy also depends on temperature alone at such states:

$$h = u + pv = u(T) + RT \quad (\text{Eq. 3.37})$$

- ▶ Collecting results, a **gas modeled as an ideal gas adheres to Eqs. 3.32-3.35 and Eqs. 3.36 and 3.37**.

## Introducing the Ideal Gas Model

- ▶ While the **ideal gas model does not provide an acceptable approximation generally**, in the **limiting case** considered in the discussion of the **compressibility chart**, it is **justified for use**, and indeed commonly applied in engineering thermodynamics at such states.
- ▶ **Appropriateness of the ideal gas model can be checked** by locating states under consideration on one of the generalized compressibility charts provided by appendix figures **Figs. A-1** through **A-3**.

## Internal Energy and Enthalpy of Ideal Gases

- ▶ For a gas obeying the **ideal gas model**, **specific internal energy depends only on temperature**. Hence, the specific heat  $c_v$ , defined by **Eq. 3.8**, is also a function of temperature alone. That is,

$$c_v(T) = \frac{du}{dT} \quad (\text{ideal gas}) \quad \text{(Eq. 3.38)}$$

- ▶ On integration,

$$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T) dT \quad (\text{ideal gas}) \quad \text{(Eq. 3.40)}$$

## Internal Energy and Enthalpy of Ideal Gases

- ▶ Similarly, for a gas obeying the **ideal gas model**, **specific enthalpy depends only on temperature**. Hence, the specific heat  $c_p$ , defined by **Eq. 3.9**, is also a function of temperature alone. That is,

$$c_p(T) = \frac{dh}{dT} \quad (\text{ideal gas}) \quad \text{(Eq. 3.41)}$$

- ▶ On integration,

$$h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T) dT \quad (\text{ideal gas}) \quad \text{(Eq. 3.43)}$$

## Internal Energy and Enthalpy of Ideal Gases

- ▶ In applications where the specific heats are modeled as constant,

$$u(T_2) - u(T_1) = c_v[T_2 - T_1] \quad (\text{Eq. 3.50})$$

$$h(T_2) - h(T_1) = c_p[T_2 - T_1] \quad (\text{Eq. 3.51})$$

- ▶ For several common gases, evaluation of changes in specific internal energy and enthalpy is facilitated by use of the *ideal gas tables*:  
**Tables A-22** and **A-23**.
- ▶ **Table A-22** applies to air modeled as an ideal gas.

# Internal Energy and Enthalpy of Ideal Gases

- ▶ **Example:** Using **Table A-22**, determine the change in specific enthalpy, in kJ/kg, for a process of air from an initial state where  $T_1 = 300 \text{ K}$ ,  $p_1 = 1 \text{ bar}$  to a final state where  $T_2 = 1500 \text{ K}$ ,  $p_2 = 10 \text{ bar}$ .
- ▶ **Solution:**  $h_1 = 300.19 \text{ kJ/kg}$ ;  $h_2 = 1635.97 \text{ kJ/kg}$

$$h_2 - h_1 = 1335.78 \text{ kJ/kg}$$

Over such a wide temperature interval, use of  $h_2 - h_1 = c_p[T_2 - T_1]$ , **Eq. 3.51**, would not be appropriate.

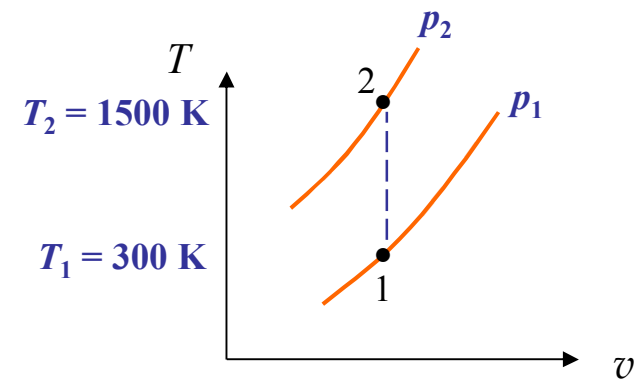
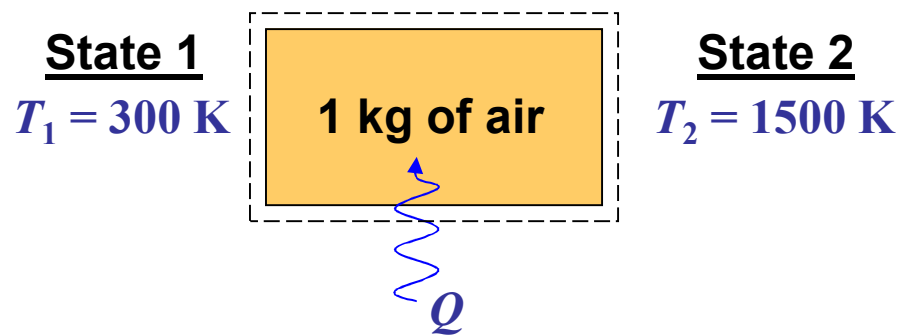
**TABLE A-22**

**Ideal Gas Properties of Air**

$T(\text{K}), h \text{ and } u(\text{kJ/kg}), s^\circ (\text{kJ/kg}\cdot\text{K})$											
$T$	$h$	$u$	$s^\circ$	when $\Delta s = 0$		$T$	$h$	$u$	$s^\circ$	when $\Delta s = 0$	
				$p_r$	$v_r$					$p_r$	$v_r$
250	250.05	178.28	1.51917	0.7329	979.	1400	1515.42	1113.52	3.36200	450.5	8.919
260	260.09	185.45	1.55848	0.8405	887.8	1420	1539.44	1131.77	3.37901	478.0	8.526
270	270.11	192.60	1.59634	0.9590	808.0	1440	1563.51	1150.13	3.39586	506.9	8.153
280	280.13	199.75	1.63279	1.0889	738.0	1460	1587.63	1168.49	3.41247	537.1	7.801
285	285.14	203.33	1.65055	1.1584	706.1	1480	1611.79	1186.95	3.42892	568.8	7.468
290	290.16	206.91	1.66802	1.2311	676.1	1500	1635.97	1205.41	3.44516	601.9	7.152
295	295.17	210.49	1.68515	1.3068	647.9	1520	1660.23	1223.87	3.46120	636.5	6.854
300	300.19	214.07	1.70203	1.3860	621.2	1540	1684.51	1242.43	3.47712	672.8	6.569
305	305.22	217.67	1.71865	1.4686	596.0	1560	1708.82	1260.99	3.49276	710.5	6.301
310	310.24	221.25	1.73498	1.5546	572.3	1580	1733.17	1279.65	3.50829	750.0	6.046

## Property Data Use in the Closed System Energy Balance

**Example:** A closed, rigid tank consists of **1 kg** of **air** at **300 K**. The air is heated until its temperature becomes **1500 K**. Neglecting changes in kinetic energy and potential energy and modeling air as an ideal gas, determine the heat transfer, in kJ, during the process of the air.



# Property Data Use in the Closed System Energy Balance

**Solution:** An energy balance for the closed system is

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - \cancel{W}$$

where the kinetic and potential energy changes are neglected and  $W = 0$  because there is no work mode.

Thus  $Q = m(u_2 - u_1)$

Substituting values for specific internal energy from **Table A-22**

$$Q = (1 \text{ kg})(1205.41 - 214.07) \text{ kJ/kg} = 991.34 \text{ kJ}$$

**TABLE A-22**

Ideal Gas Properties of Air

<i>T(K), h and u(kJ/kg), s° (kJ/kg·K)</i>											
<i>T</i>	<i>h</i>	<i>u</i>	<i>s°</i>	when $\Delta s = 0$		<i>T</i>	<i>h</i>	<i>u</i>	<i>s°</i>	when $\Delta s = 0$	
				<i>p<sub>r</sub></i>	<i>v<sub>r</sub></i>					<i>p<sub>r</sub></i>	<i>v<sub>r</sub></i>
250	250.05	178.28	1.51917	0.7329	979.	1400	1515.42	1113.52	3.36200	450.5	8.919
260	260.09	185.45	1.55848	0.8405	887.8	1420	1539.44	1131.77	3.37901	478.0	8.526
270	270.11	192.60	1.59634	0.9590	808.0	1440	1563.51	1150.13	3.39586	506.9	8.153
280	280.13	199.75	1.63279	1.0889	738.0	1460	1587.63	1168.49	3.41247	537.1	7.801
285	285.14	203.33	1.65055	1.1584	706.1	1480	1611.79	1186.95	3.42892	568.8	7.468
290	290.16	206.91	1.66802	1.2311	676.1	1500	1635.97	1205.41	3.44516	601.9	7.152
295	295.17	210.49	1.68515	1.3068	647.9	1520	1660.23	1223.87	3.46120	636.5	6.854
300	300.19	214.07	1.70203	1.3860	621.2	1540	1684.51	1242.43	3.47712	672.8	6.569
305	305.22	217.67	1.71865	1.4686	596.0	1560	1708.82	1260.99	3.49276	710.5	6.301
310	310.24	221.25	1.73498	1.5546	572.3	1580	1733.17	1279.65	3.50829	750.0	6.046



# Polytropic Process

- ▶ A polytropic process is a quasiequilibrium process described by

$$pV^n = \text{constant} \quad (\text{Eq. 3.52})$$

- ▶ The exponent,  $n$ , may take on any value from  $-\infty$  to  $+\infty$  depending on the particular process.
  - ▶ For **any gas (or liquid)**, when  $n = 0$ , the process is a constant-pressure (**isobaric**) process.
  - ▶ For **any gas (or liquid)**, when  $n = \pm\infty$ , the process is a constant-volume (**isometric**) process.
  - ▶ For a gas modeled as an **ideal gas**, when  $n = 1$ , the process is a constant-temperature (**isothermal**) process.