FORMULATIONS FOR SCHOOL BUS ROUTING PROBLEMS

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OUTLINE

- Problem Definition
- Components of the Problem
- Solution Methods
- Integer Programming Models
- Computational Results
- Conclusions and Future Directions
Problem Definition

In School Bus Routing Problem, the students are picked up from gathering points and brought to school or vice versa by minimizing total transportation costs.

Components of the Problem

Decision Makers

School Administrators
Parents
Bus Firms
Local Authorizes
Components of the Problem

Parameters

- The number of students to be picked up or dropped off.
- The distance between the points.
- The number of seats available on the bus for each one.
- The maximum time/distance limits for each route.
- The operating cost for each bus.
- ...

Objectives

- Minimizing the operating cost with reducing the number of buses.
- Minimizing the number of bus routes.
- Minimizing the total bus traveling time.
- Balancing the load of each bus or traveling distance for each bus.
- Minimizing the student walking time to the bus stops.
- ...

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Components of the Problem

Constraints

- The number of students must not exceed the number of seats on each bus.
- The distance/time for traveling on the bus must not exceed a given limit.
- Each bus stop is allocated to only one bus.
- Each stop is allocated to only one route.
- Every route must have at least one stop.
- ...

Solution Methods

- Formulations and/or Exact Methods
- Heuristic Approaches
  - Tabu Algorithm
  - Sweep Algorithm
  - Harmony Search
  - Improvement Algorithms
  - ....

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Motivation

In real life applications, problem can not be defined on the symmetric distance matrix, because of the traffic situation in urban areas. Therefore, the morning and afternoon problems can not be solved with same model.

Modeling Phases?

I. Đ. KARA VRP Formulations
II. Adaptation
III. Asymmetrical Situation
IV. Morning / Afternoon routes

Notations - 1/2

Sets

M = {1,2,...,m} The set of all busses
V : { 2, 3,..., n } The set of all bus stops
V′ : {1, 2, 3...,n, n+1} All routing points
1→ School (n+1) → Depot

Parameters

D_{ij} = the distance required to traverse from node i to node j
C_{ij} = the cost required to traverse from node i to j
C_{ij} = αD_{ij} (α: is a constant which changing the distance matrix to cost matrix)
Q = bus capacity
D = maximum distance limit
f = the fixed cost per a bus
Decision Variables

\( X_{ij} = 1 \), if \((i,j)\) exists in the solution; 0, otherwise

\( m \) = the number of required bus (also shows that required tours)

General Formulation

\[
\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} + \text{fm}
\]

Subject To

\[
\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \in V
\]

\[
\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j \in V
\]

\[
\sum_{i=2}^{n} x_{i+1,i}, \leq m \quad \sum_{i=2}^{n} x_{i,1} \leq m \quad \text{(For morning routes)}
\]

\[
\sum_{i=2}^{n} x_{ii} \leq m \quad \sum_{i=2}^{n} x_{i,i+1} \leq m \quad \text{(For afternoon routes)}
\]

Sub tour Elimination Constraints
+ Capacity Constraints
+ Distance Constraints
**Node Based Formulations**

**NF Models**

**Morning Routes (NF1)**

- $u_i$: the total amount of students picked up by the vehicle just after node $i$.
- $\nu_i$: the total length traveled from the depot to node $i$ ($i \in V$).

**Afternoon Routes (NF2)**

- $u_i$: the total amounts of students dropped off by the vehicle just after node $i$.
- $\nu_i$: the total length traveled from the school to node $i$ ($i \in V$).

---

**Node Based Formulation (NF1) Morning Routes (Collection)**

\[
\begin{align*}
\min & \sum_{j=1}^{n} \sum_{i=2}^{n} c_{ij} x_{ij} + fm \\
\text{Subject to} & \\
\sum_{i=2}^{n} x_{n+1,i} & \leq m \quad \text{only m buses can leave from depot} \\
\sum_{i=2}^{n} x_{i,1} & \leq m \quad \text{only m buses can return to school} \\
\sum_{j=1}^{n} x_{ij} & = 1 \quad \forall i \in V \quad \text{degree constraints} \\
\sum_{i=2}^{n+1} x_{ij} & = 1 \quad \forall j \in V \quad \text{each stop is visited exactly once}
\end{align*}
\]
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Node Based Formulation (NF2)
Afternoon Routes (Delivery)

\[
\begin{align*}
\min \sum \sum c_{ij} x_{ij} + fm \\
\text{Subject to} \\
\sum_{i=2}^{n} x_{1i} &\leq m \quad \text{only m buses can leave from school} \\
\sum_{i=2}^{n} x_{i,n+1} &\leq m \quad \text{only m buses can return to depot} \\
\sum_{j=2}^{n+1} x_{ij} & = 1 \quad \forall i \in V \quad \text{degree constraints} \\
\sum_{i=1}^{n} x_{ij} & = 1 \quad \forall j \in V \quad \text{each stop is visited exactly once}
\end{align*}
\]

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Flow Based Formulations

**FF Models**

**Morning-Afternoon Routes**

- \( y_{ij} \): the total amounts of students on the vehicle between node \( i \) and \( j \)
- \( t_{ij} \): the total length traveled from the depot(school) to node \( j \).
Flow Based Formulation (FF1)  
Morning Routes (Collection)

\[
\begin{align*}
\text{min} & \sum_{j=2}^{n} \sum_{i=1}^{n} c_{ij} x_{ij} + f_{in} \\
\text{Subject to} & \\
\sum_{i=2}^{n} x_{i+1,j} \leq m & \quad \text{only m buses can leave from depot} \\
\sum_{i=2}^{n} x_{i,1} \leq m & \quad \text{only m buses can return to school} \\
\sum_{j=1}^{n} x_{ij} = 1 & \quad \forall i \in V \quad \text{degree constraints} \\
\sum_{i=2}^{n+1} x_{ij} = 1 & \quad \forall j \in V \quad \text{each stop is visited exactly once}
\end{align*}
\]
Flow Based Formulation (FF2)

Afternoon Routes (Delivery)

\[
\begin{align*}
\text{min} & \sum c_{ij}x_{ij} + \text{fm} \\
\text{Subject to} & \\
\sum_{i=2}^{n} x_{1i} & \leq m \quad \text{only m buses can leave from school} \\
\sum_{i=2}^{n} x_{i,n+1} & \leq m \quad \text{only m buses can return to depot} \\
\sum_{j=2}^{n+1} x_{ij} & = 1 \quad \forall i \in V \quad \text{degree constraints} \\
\sum_{i=1}^{n} x_{ij} & = 1 \quad \forall j \in V \quad \text{each stop is visited exactly once} \\
\end{align*}
\]

Subject to

Assignment

Capacity

Distance

\[
\begin{align*}
\sum_{j=1}^{n} y_{j} - \sum_{j=2}^{n} y_{j} & = q_{i} \quad i = 2,3,\ldots, n \\
\sum_{j=1}^{n} y_{j} & = \sum_{j=1}^{n} q_{j} \\
y_{i} & \leq (Q - q_{i})x_{ij} \quad i = 1,\ldots,n; j = 2,\ldots,n \\
y_{ij} & \geq q_{j}x_{ij} \quad i = 1,\ldots,n; j = 2,\ldots,n \\
\sum_{j=1}^{n} t_{ij} - \sum_{j=2}^{n} t_{ij} - \sum_{j=2}^{n} d_{ij}x_{ij} & = 0 \quad i = 2,3,\ldots, n \\
t_{ij} & \leq (D) x_{ij} \quad i = 2,\ldots,n; j = 1,\ldots,n+1 \\
t_{ij} & \geq (d_{ij}) x_{ij} \quad i = 2,\ldots,n; j = 2,\ldots,n \\
t_{ij} & = d_{ij}x_{ij} \quad i = 2,3,\ldots,n \\
x_{ij} & \in \{0,1\}, \forall (i,j) \\
x_{ij} & \geq 0 \quad t_{ij} \geq 0, i, j \in V \\
\end{align*}
\]
Computational Results

Computational results was done for asymmetrical problems.
- 30 asymmetrical problems are solved by generating randomly
- Tests were performed on a Intel® Pentium®4 CPU 3.00 Ghz, 3.04 Ghz and 2.00 GB RAM
- CPLEX 10.0.0 was used to solve the models.

Asymmetric Problems

- Bus stops: 13 + Depot + School
- \( C_{ij} \sim [0,1000] \): The cost of traveled distance was chosen uniformly between 0 and 1000.
- \( q_i \) [1,24]: students are assigned to the bus stops uniformly between 1 and 24.
- \( Q \) (capacity) (25): all busses have 25 seats capacity
- \( D \) (maximum distance): all tours can not exceed the maximum distance limit
  \[ 2 \times \frac{N - 1}{\sum q_i} \times 0.7 \times \text{Max}(d_i) \]
- \( f \) (fixed cost per bus):
### Asymmetric Problems—CPU Times

<table>
<thead>
<tr>
<th></th>
<th>Problem (29/30)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average (sec)</td>
<td>463.9</td>
<td>9.3</td>
</tr>
<tr>
<td>Std Dev</td>
<td>1014.65</td>
<td>35.9</td>
</tr>
<tr>
<td>Maximum</td>
<td>3370.3</td>
<td>184.3</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.03</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Afternoon</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average (sec)</td>
<td>345.0</td>
<td>11.0</td>
</tr>
<tr>
<td>Std Dev</td>
<td>995.7</td>
<td>43.4</td>
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<tr>
<td>Maximum</td>
<td>4458.6</td>
<td>223.6</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.02</td>
<td>1.0</td>
</tr>
</tbody>
</table>

CPU time limit: 7200 sec.

### Conclusions—Future Directions—1/2

- The SBRP is a real life problem.
  - The main objective is minimizing the total cost of traveling.
  - Capacity and distance constraints are important and essential issues.
  - We investigated that type of constraints.

- The models have polynomial size $O(n^2)$ constraints and decision variables for each model.
  - This is a different modeling approach for SBRP for small and medium size problems.
  - Developing computer technology, we would assume to solve large size problems with these models.
Conclusions-Future Directions-2/2

- Flow based formulations give quicker solutions than the node based formulations.
- Flow based formulations produce better LPR than the node based formulations.
- Load/length balancing
- We propose utilization of our flow based formulation to the researchers who want to develop model based exact and/or heuristic algorithms.

Thank you for listening.


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