

# Short-time Fourier Transform (STFT) and Spectrogram

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## Short-time Fourier Transform

- The **short-time Fourier transform (STFT)**, or alternatively **short-term Fourier transform**, is a [Fourier-related transform](#) used to determine the sinusoidal frequency and phase content of local sections of a signal as it changes over time.

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## STFT

- The function to be transformed is multiplied by a window function which is nonzero for only a short period of time
- The [Fourier transform](#) of the resulting signal is taken as the window is slid along the time axis, resulting in a two-dimensional representation of the signal.
- $w(t)$  is [window function](#), commonly a [Hann window](#) or [Gaussian](#) "hill" centered around zero

$$\text{STFT } \{x(t)\} \equiv X(\tau, \omega) = \int_{-\infty}^{\infty} x(t)w(t - \tau)e^{-j\omega t} dt$$

## Discrete STFT

- Data to be transformed could be broken up into chunks or frames (which usually overlap each other).
- Each chunk is [Fourier transformed](#), and the complex result is added to a matrix, which records magnitude and phase for each point in time and frequency.

$$\text{STFT } \{x[n]\} \equiv X(m, \omega) = \sum_{n=-\infty}^{\infty} x[n]w[n - m]e^{-j\omega n}$$

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## Spectrogram

- The **spectrogram** is the result of calculating the [frequency spectrum](#) of [windowed frames](#) of a compound [signal](#). It is a three-dimensional plot of the energy of the frequency content of a signal as it changes over time.
- Spectrograms are used to identify [phonetic](#) sounds, to analyse the cries of animals, and in the fields of [music](#), [sonar/radar](#), [speech processing](#), etc.

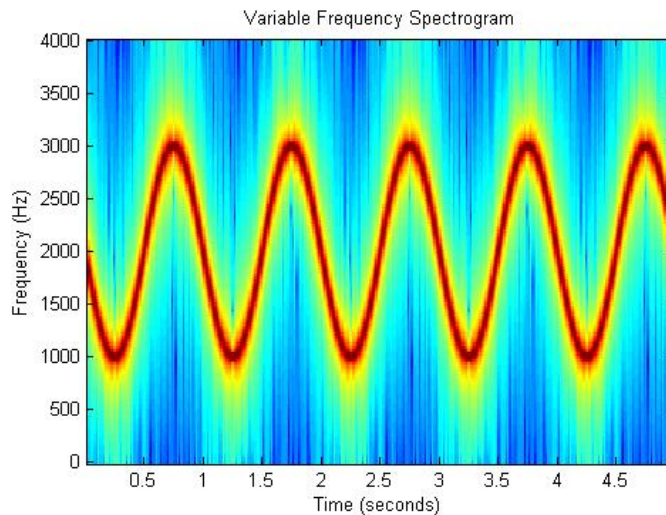
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## Spectrogram Format

- In the most usual format, the horizontal axis represents [time](#), the vertical axis is [frequency](#), and the [intensity](#) of each point in the image represents amplitude of a particular frequency at a particular time.

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## A spectrogram of an FM signal.



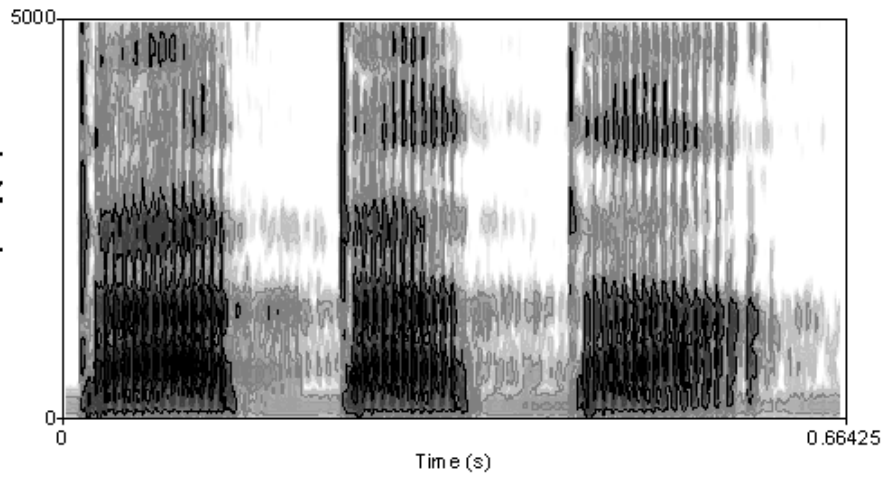
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## Spectrogram Calculation

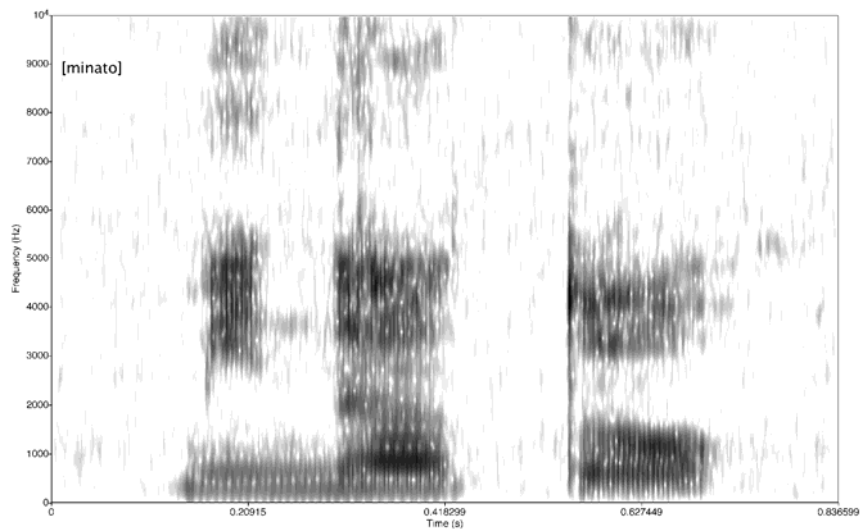
- Spectrograms are usually calculated from the time signal using the [short-time Fourier transform](#) (STFT).
- Digitally [sampled](#) data, in the time domain, is broken up into chunks, which usually overlap
- Fourier transformed to calculate the magnitude of the frequency spectrum for each chunk.
- Each chunk then corresponds to a vertical line in the image; a measurement of magnitude versus frequency for a specific moment in time.
- The spectrums or time plots are then "laid side by side" to form the image or a three-dimensional surface.

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## Spectrogram of a male voice saying "tatata"



## Spectrogram of a Japanese woman saying "minato"



## Pattern Classification

- Supervised Pattern Classification
  - We are provided with a number of feature vectors with classes assigned to them. These are the techniques to characterize the boundaries that separate the classes.
- Unsupervised Pattern Classification
  - We are given a set of feature vectors with no categorization or classes attached to them. No prior training information is available.

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## Application: Normal versus Ectopic ECG Beats

1. Proper filtering of ECG
2. Pan-Tompkins to detect each beat
3. Select QRS-T interval from the sample 160 ms before the peak of the Pan-Tompkins output to the sample 240 ms after the peak
4. Compute RR interval and FF and use them as the features
5. Find mean of normal and ectopic beats
6. Equations of the straight line joining the two prototype vectors and its normal bisector were determined
  - Optimal decision function
7.  $RR - 5.56FF + 11.44 = 0$  decision function
8. If  $RR - 5.56FF + 11.44 > 0$  normal beat  
     $RR - 5.56FF + 11.44 \leq 0$  ectopic beat

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