Chapter 7
Modeling Biomedical Signal-Generating Processes and Systems

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Mathematical modeling

- In modeling approach, an explicit mathematical model is used to represent the process or the system that generates the signal of interest.
- The parameters of the model are then investigated for use in signal analysis, pattern recognition, and decision making.
- Model parameters may also be related to physical aspects of the related systems.

Problem statement:

- Propose mathematical models to represent the generation of biomedical signals.
- Identify the possible relationship between the mathematical models and the physiological and pathological processes and systems that generate the signals.
- Explore the potential use of the model parameters in signal analysis, pattern recognition, and diagnostic decision making.

Case study 1: Motor-unit firing patterns

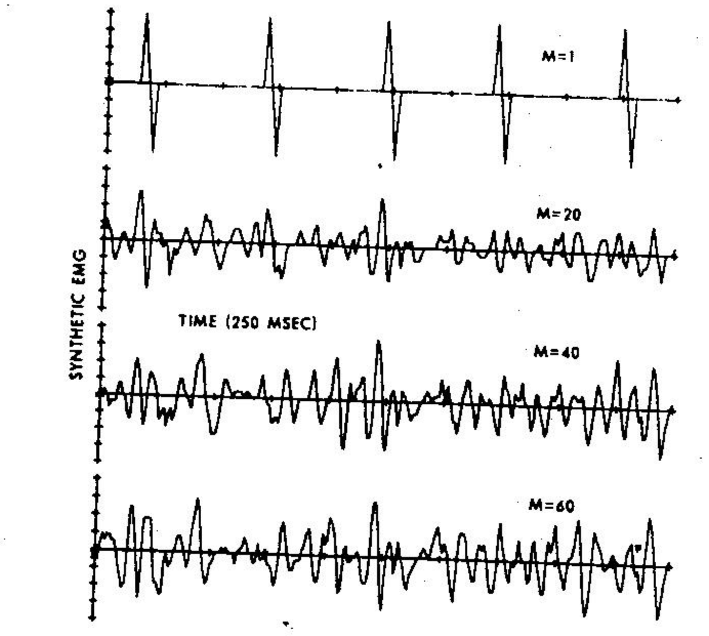
- Surface EMG signal of an active skeletal muscle
  - Spatio-temporal summation of the action potentials of a large number of motor units that have been recruited into action
- EMG of a single motor unit
  - Train of SMUAP’s (Single Motor Unit Action Potentials)
  - Same basic wave (spike, pulse or wavelet) is repeated in a period

Train of SMUAP’s

- We may represent the intervals between the SMUAP’s by a random variable.
- Although an overall periodicity exists the intervals between pulses (firing rate in pps), known as inter-pulse-interval (IPI) may not be precisely the same from one SMUAP to another.

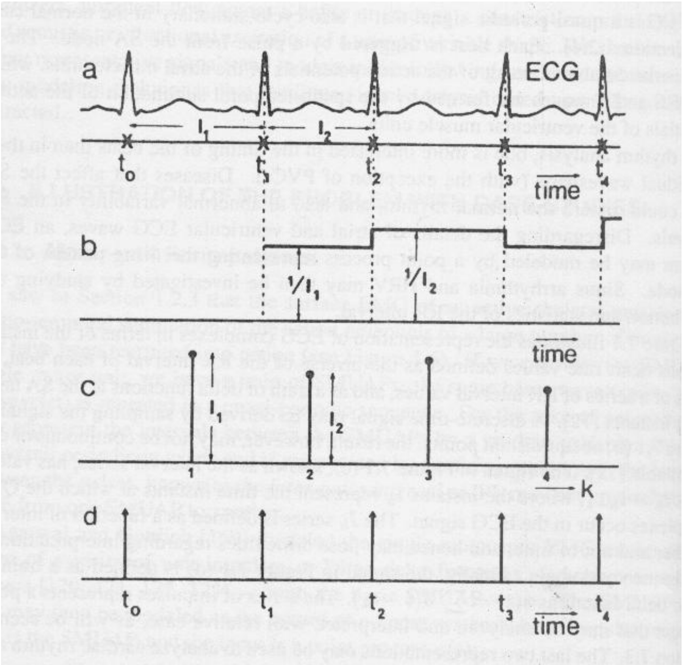
Modeling

- Single motor unit EMG can be modeled as the convolution of a series of unit impulses or Dirac delta functions (point process) with the basic SMUAP wave
  - $y(t) = x(t)*h(t)$
- Constraints:
  - From the same motor unit successive action potentials cannot overlap
  - The interval between any two pulses should be greater than the SMUAP duration
  - SMUAP duration=3-20 ms, firing rate 7-25 pps (pulse per second) -> IPI 40-140 ms
  - An SMUAP train consists of distinct and separated events waves



Case study 2: Cardiac rhythm

- In rhythm analysis one is more interested in the timing of the beats than in their individual waveshape
- Sinus arrhythmia and HRV (heart rate variability) may be investigated by studying the distribution and statistics of the RR interval.
- Interval series: $I_k = t_k - t_{k-1}$
  - The instants t_k represent the time instants at which the QRS complexes occur in the ECG signal
  - I_k is a function of interval number not of time
  - $s(t) = \Sigma\ \delta\ (t - t_k)$ -> a function of time



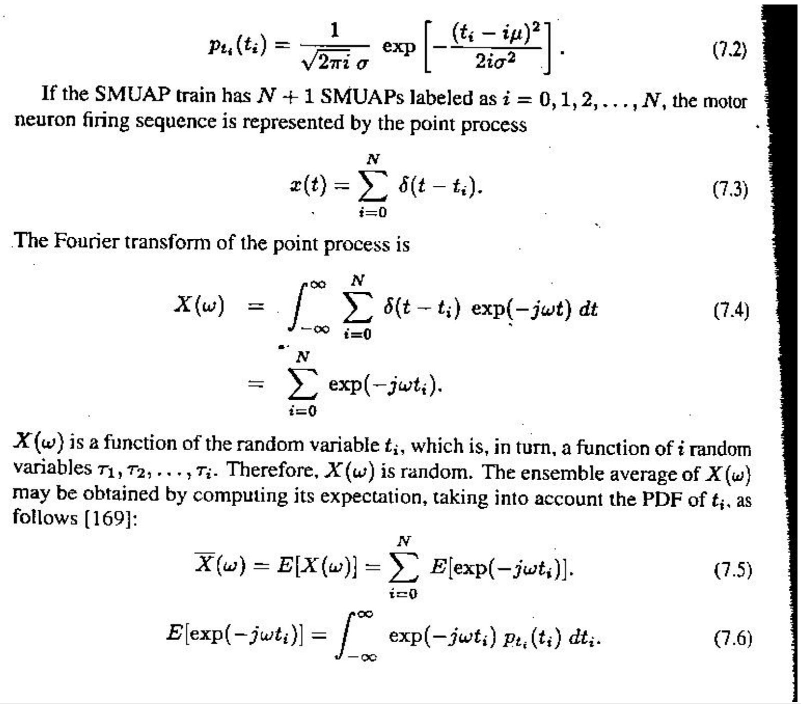
ure 7.1 The train of ECG complexes in (a) is represented in terms of

Point process

- A point process is a special type of stochastic process, where the time points of occurrences of events are stochastic.
- Counting process: Number of events in a fixed time interval
- Interval process: The time intervals between subsequent events

Point processes

- Let the interval between the i'th SMUAP and the preceding one be τ_i and let the origin be set at the instant of appearance of the first SMUAP at i=0 with $\tau_0 = 0$
- The time of arrival of the i'th SMUAP is then given by $t_i = \tau_1 + \tau_2 + \dots \tau_i$
- The variable t_i is the sum of i independent random variables
  - Mean and variance of the random variable representing each IPI are the same



$$p_{t_i}(t_i) = \frac{1}{\sqrt{2\pi i}\,\sigma}\ \exp\left[-\frac{(t_i - i\mu)^2}{2i\sigma^2}\right]. \tag{7.2}$$

If the SMUAP train has $N+1$ SMUAPs labeled as $i = 0, 1, 2, \dots, N$, the motor neuron firing sequence is represented by the point process

$$x(t) = \sum_{i=0}^{N} \delta(t - t_i). \tag{7.3}$$

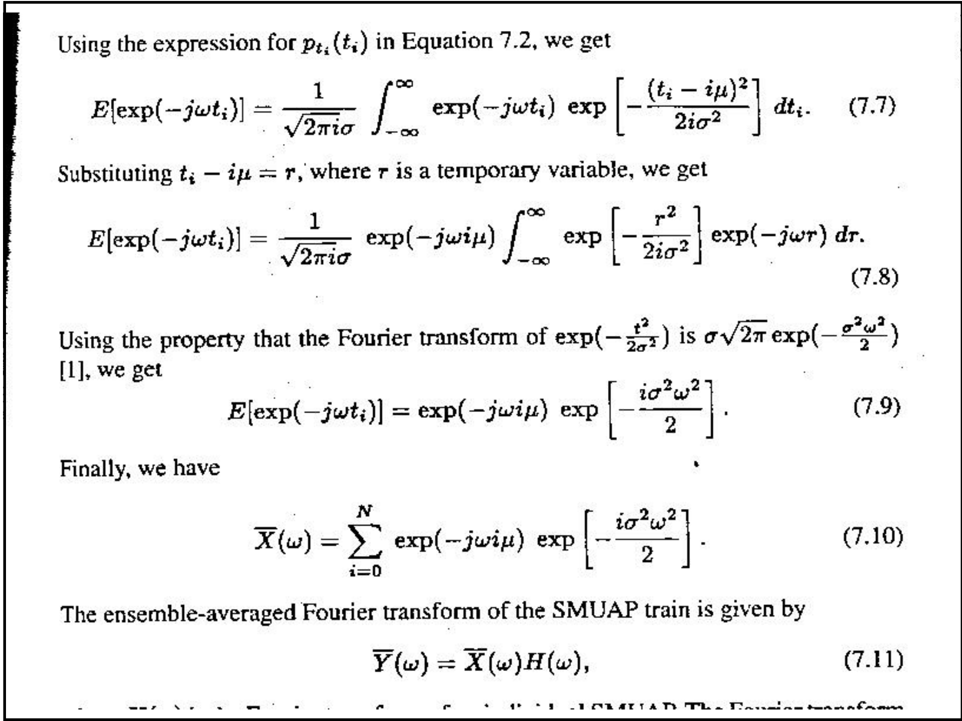
The Fourier transform of the point process is

$$\begin{aligned} X(\omega) &= \int_{-\infty}^{\infty} \sum_{i=0}^{N} \delta(t - t_i)\ \exp(-j\omega t)\ dt \\ &= \sum_{i=0}^{N} \exp(-j\omega t_i). \end{aligned} \tag{7.4}$$

$X(\omega)$ is a function of the random variable t_i, which is, in turn, a function of i random variables $\tau_1, \tau_2, \dots, \tau_i$. Therefore, $X(\omega)$ is random. The ensemble average of $X(\omega)$ may be obtained by computing its expectation, taking into account the PDF of t_i, as follows [169]:

$$\overline{X}(\omega) = E[X(\omega)] = \sum_{i=0}^{N} E[\exp(-j\omega t_i)]. \tag{7.5}$$

$$E[\exp(-j\omega t_i)] = \int_{-\infty}^{\infty} \exp(-j\omega t_i)\ p_{t_i}(t_i)\ dt_i. \tag{7.6}$$



Using the expression for $p_{t_i}(t_i)$ in Equation 7.2, we get

$$E[\exp(-j\omega t_i)] = \frac{1}{\sqrt{2\pi i}\,\sigma} \int_{-\infty}^{\infty} \exp(-j\omega t_i)\ \exp\left[-\frac{(t_i - i\mu)^2}{2i\sigma^2}\right] dt_i. \quad (7.7)$$

Substituting $t_i - i\mu = r$, where r is a temporary variable, we get

$$E[\exp(-j\omega t_i)] = \frac{1}{\sqrt{2\pi i}\,\sigma}\ \exp(-j\omega i\mu) \int_{-\infty}^{\infty} \exp\left[-\frac{r^2}{2i\sigma^2}\right] \exp(-j\omega r)\ dr. \quad (7.8)$$

Using the property that the Fourier transform of $\exp(-\frac{t^2}{2\sigma^2})$ is $\sigma\sqrt{2\pi}\exp(-\frac{\sigma^2\omega^2}{2})$ [1], we get

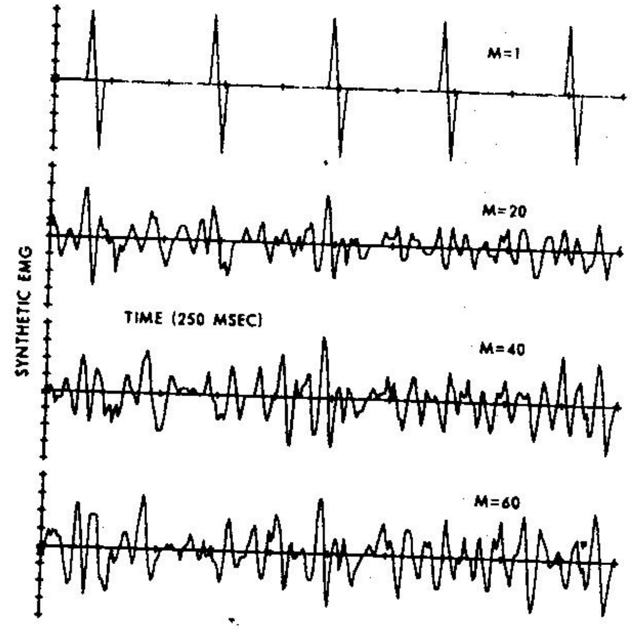
$$E[\exp(-j\omega t_i)] = \exp(-j\omega i\mu)\ \exp\left[-\frac{i\sigma^2\omega^2}{2}\right]. \quad (7.9)$$

Finally, we have

$$\overline{X}(\omega) = \sum_{i=0}^{N} \exp(-j\omega i\mu)\ \exp\left[-\frac{i\sigma^2\omega^2}{2}\right]. \quad (7.10)$$

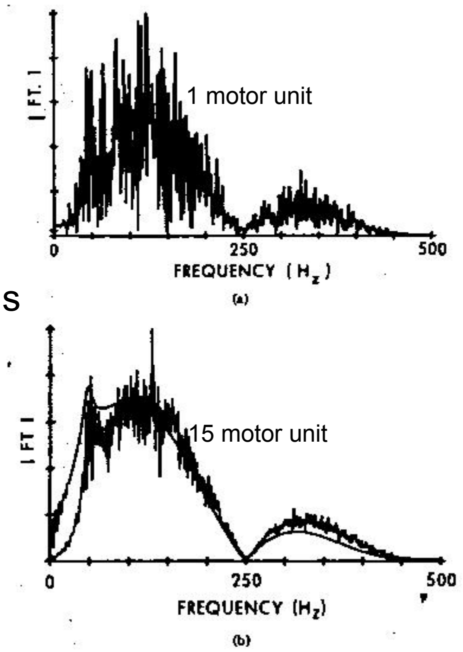
The ensemble-averaged Fourier transform of the SMUAP train is given by

$$\overline{Y}(\omega) = \overline{X}(\omega) H(\omega), \quad (7.11)$$

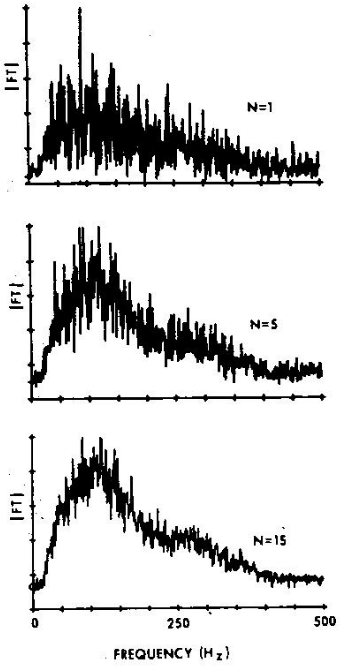


SMUAP Duration = 8 ms μ = 50 ms and σ = 6.27 ms

SMUAP Dur = 8 ms
μ = 50 ms, σ = 4.36 ms



Magnitude spectra of surface
EMG signals recorded from
Gastrocnemius-soleus muscle
Over 1, 5, and 15 signal records.



Parametric System Modeling

- Problem: Explore the possibility of parametric modeling of signal characteristics using the general linear system model
- $y[n] = - \sum_{k=1}^{P} a_k \, y[n-k] + G \sum_{l=0}^{Q} b_l \, x[n-l]$

Output combination

How the present and past
Q samples of the input are combined

G: Gain factor, P and Q determine the order of the system

Parametric System Modeling

- $y[n] = - \Sigma \, a_k \, y[n-k] + G \, \Sigma \, b_l \, x[n-l]$
- Summation over x represents the MOVING AVERAGE (MA)
- Summation over y represents the AUTOREGRESIVE (AR)
- Entire system ARMA -> IIR filter
- The model indicates that the present output sample may be predicted as a linear combination of the present and a few past input samples and a few past output samples
  - Linear prediction (LP) model

LP model

- $H(z) = Y(z)/X(z) = G\ (1 + \sum_{l=0}^{Q} b_l\ z^{-l})/(1 + \sum_{k=1}^{P} a_k\ z^{-k})$
- The system is completely characterized by a and b parameters
- Same conceptual model in both the time and the frequency domains is applicable expressing /o relationship or the system transfer function

Autoregressive Modeling

- Methods to determine an AR model for a given signal for which the corresponding input to the system is not known
- $y[n] = - \sum_{k=1}^{P} a_k\ y[n-k] + G\ x[n]$
- $H(z) = Y(z)/X(z) = G/(1 + \Sigma a_k\ z^{-k})$
- Predicted value $y_p[n]$ is only an approximation
- Error = y[n] – yp[n]
- a_k's are obtained by minimizing mean squared error with respect to all of the parameters