Chapter 6
Frequency-Domain Characterization of Signals and Systems

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Introduction

- In many biomedical systems frequency is used instead of time:
  - Cardiac rhythm: Beats per minute rather than RR interval
  - EEG rhythm: alpha waves, frequency 10 Hz rather than 0.1 sec period

PCG example

- There is beat-to-beat periodicity
- Due to multi-compartmental nature of the cardiac system
  - Heart sounds possess multiple resonance frequencies
- PCG should be described both with a rhythm (heart rate) or a single resonance frequency also a composite spectrum of several dominant or resonance frequencies

PCG example cont'd

- Septal defect or stenosed valve -> turbulance -> wide-band noise
- In case of noise-like murmurs, we cannot identify rhythms or resonance frequencies
- We need to consider distribution of the signal's energy or power over a wide band of frequencies -> PSD function

Goal

- Investigate methods to estimate the PSD and frequency-domain parameter of biomedical signals and systems
- Distinguish between normal and abnormal signal or systems. This has potential in the diagnosis.

Problem statement

- Investigate the potential use of Fourier spectrum and parameters derived thereof in the analysis of biomedical signals.

PCG components

- S1 and S2 have low-frequency components
  - Due to fluid-filled and elastic nature of the cardiohemic system
- Heart sound spectra is max in the 20-40 Hz range
  - S1 demonstrates peaks and lower frequency than those of S2
  - S2 has a gentle peaking between 60-220 Hz

Case-study I: The effect of myocardial elasticity on heart sound spectra

- Frequency content of S1 during iso-volumetric contraction period should depend on the relative contributions of the mass and elasticity of the left ventricle (LV)
- Mass of LV is constant, so frequency content of S1 should decrease in the case of diseases that reduce ventricular elasticity, such as MI.

- How can we determine the existence of myocardial infarct (MI) using PCG?

Propose hardware and software solutions:

- Hardware methods:
  1. Tunable bandpass filters (20-40, 40-60, … ,400-420 Hz) -> frequency distribution of S1 and S2.
     – Determine which filter yields the highest S1 in voltage (20-40 Hz)
     – Do the same for S2
     – On average max S1 ~40 Hz for normal, for MI patients this is lower
  2. Usage of dynamic spectrum analyzer to study the frequency content of S1 during the isovolumetric contraction period
     – Similar procedure

Software methods:

1. Simulate filter bank (20-40, 40-60, … 400-420 Hz)
   - Obtain averaged power spectra on people
   - Average PSD should be -10 dB beyond 150 Hz
2. Apply FFT for the analysis of S1 and S2
   - FFT spectra of 250 ms windows containing S1 are averaged over 15 beats for each subject

Case-study II: Valvular problems

- Cardiovascular defects and diseases cause high-frequency noise-like murmurs
- ZCR can be used, but a good result is not obtained
- PSD of S1 and S2
  - Find the timing of high-frequency components in the PCG signal
  - On average last 1/3 of the systole, murmurs are detected (observations show this fact).

The Fourier Spectrum

- To measure the amplitude and phase of a particular frequency component, the transform process multiplies the original function (the one being analyzed) by a sinusoid with the same frequency (called a basis function). If the original function contains a component with the same shape (i.e. same frequency), its shape (but not its amplitude) is effectively squared.
- To make that happen actually requires two sinusoidal basis functions, cosine and sine, which are combined into a basis function that is complex-valued (Complex exponential). The vector analogy refers to the polar coordinate representation.
- The complex numbers produced by the product of the original function and the basis function are subsequently summed into a single result.
- The contributions from the component that matches the basis function all have the same sign (or vector direction). The other components contribute values that alternate in sign (or vectors that rotate in direction) and tend to cancel out of the summation. The final value is therefore dominated by the component that matches the basis function. The stronger it is, the larger is the measurement. Repeating this measurement for all the basis functions produces the frequency-domain representation.

The Fourier Spectrum

$$S(f) = \int_{-\infty}^{\infty} s(t) \cdot e^{-i2\pi f t} dt$$

$$S_T(f) = \sum_{n=-\infty}^{\infty} s[n] \cdot e^{-i2\pi f n T} = \sum_{n=-\infty}^{\infty} s[n] \cdot e^{-i2\pi \frac{f}{f_s} n},$$

PSD estimation

- We know that Fourier Transform of ACF is PSD
- x[n], n=0,1,2, …, N-1 (finite duration signal)
- True ACF: $\Phi_{xx}(m)=E(x[n] x[n+m])$
- Definition of ACF as a statistical expectation or an integral over a duration tending to infinity
  - $\Phi_{xx}(t_1,t_1+\tau)=\lim (1/M) \sum_{k=1}^{M} x_k(t_1)\, x_k(t_1+\tau)$ (for M goes to inf)
  - $\Phi_{xx}(\tau,k)=\lim (1/T) \int_{-T/2}^{T/2} x_k(t)\, x_k(t+\tau)\, dt$ (for T goes to inf)
- $\Phi_{xx}(0)= x[n] x[n] = x[n]^2$, from n=0 to N-1
- $\Phi_{xx}(1)= x[n] x[n+1]$, from n=0 to N-2 that is N-|m| multiplications and summations

PSD estimation

- One type of averaging
- $\Phi_1(m) = (1/ N-|m|) \sum_{n=0}^{N-|m|-1} x[n]\, x[n+m]$ is a good estimate for $\Phi_{xx}(m)$
- For m close to N, estimate is useless
- $\Phi_2(m) = (1/N) \sum_{n=0}^{N-|m|-1} x[n]\, x[n+m]$, for all delays same scaling factor is used
- $\Phi_2(m) = (N-|m| / N)\, \Phi_1(m)$

PSD estimation

- ACF $\xrightarrow{FT}$ PSD
- $S_2(w) = \sum_{m=-(N-1)}^{N-1} \Phi_2(m)\, e^{(-jwm)}$, delay up to N-1 is available
- $S_2(w) = (1/N)\, |X(w)|^2$ -> PSD estimate of finite length x[n] is called "periodogram"

The need for averaging

- To reduce the variance of an estimate it is necessary to average over a number of statistically independent samples (similar to synchronized averaging) to obtain a better estimate of PSD

Problem:

- Propose a method to obtain an averaged PSD estimates of the “systolic” and “diastolic” heart sounds.

Solution:

- Acquisition of heart sounds over the multiple cardiac cycles
  - Direct averaging of PCG signals could lead to undesired cancellation of noise-like murmurs or asynchronous frequency components and their disappearance from the result.
- Segmentation of S1 and S2 (simultaneous recoding with ECG and CP)
- Compute the PSD estimates of each S1 segment and S2 segment separately
- Average the PSD estimates for S1 and S2

Problem:

- How can we obtain an averaged periodogram when we are given only one signal record of finite duration?

Solution:

- Divide x[n] (n=0,1,…,N-1) into K segments of M samples each
  - $x_i[n] = x[n + (i-1) M], 0 \leq n \leq M-1, 1 \leq i \leq K$
- Compute the periodogram of each segment
  - Compute FFT of each $x_i[n]$, take the square, divide by M
- Final step: Add periodograms of K segments and divide by K

ACF estimation

- Good ACF estimates ate required in applications such as optimal Wiener filter
- Determine PSD estimate using methods described in previous slides
- Take inverse FFT of the PSD estimate
- Use the result as an estimate of the ACF